A

PRACTICAL ESSAY

ON THE

STRENGTH OF CAST IRON,

INTENDED FOR THE ASSISTANCE OF

ENGINEERS, IRON MASTERS, ARCHITECTS, MILLWRIGHTS, FOUNDERS, SMITHS, AND OTHERS ENGAGED IN THE CONSTRUCTION OF MACHINES, BUILDINGS, &c.

CONTAINING

PRACTICAL RULES, TABLES, AND EXAMPLES;

ALSO AN ACCOUNT OF SOME

NEW EXPERIMENTS,

With an Extensive Table of the

PROPERTIES OF MATERIALS.

ILLUSTRATED BY FOUR ENGRAVINGS.

BY THOMAS TREDGOLD,

CIVIL ENGINEER;

MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS; AUTHOR OF ELEMENTARY PRINCIPLES OF CARPENTRY; THE ARTICLE JOINERY IN THE SUPPLEMENT TO THE ENCYCLOPÆDIA BRITANNICA, &c.

—— "The same Truth, which is a Principle in science, becomes a Rule in art." *Playfair.*

London:

PRINTED FOR J. TAYLOR,

AT THE ARCHITECTURAL LIBRARY, 59, HIGH HOLBORN.

1822.

TO

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SCIENCES OF STOCKHOLM, &c.

THIS ESSAY

ON THE

STRENGTH OF CAST IRON

IS INSCRIBED BY

THE AUTHOR.

PREFACE.

In the following Pages, I have attempted to supply a Practical Treatise on the Strength of Cast Iron; the use and advantage of such a work will be best appreciated by those who consider the serious consequences of a failure in the application of this material. It is used for the principal supports of Churches, Theatres, Dwelling-houses, Manufactories and Warehouses; for Bridges, Roofs, and Floors; and for the moving parts of the most powerful Engines. If a failure take place, from want of strength, it will most probably happen at that moment when its consequences will be most serious; hence, I think I may venture to say, without giving any undue importance to the object of this work, that, if there be one sub-

ject which requires the aid and assistance of science more than another, it is the application of supports of Cast Iron*.

The very considerable improvements that have been made in the manufacture of iron, have, undoubtedly, chiefly arisen out of the peculiar advantages derived from its use, in the mining and manufacturing districts of Britain; and the immense quantities of it employed, in these districts, is one of the most satisfactory proofs of its utility and value.

These improvements in the manufacture of iron, have also enabled the manufacturers to reduce its price; so far indeed that it now can be employed, instead of foreign timber, for many important purposes in buildings and machines, at a very small additional expense, with a considerable addition of soundness and durability. It is not, however, fitted for every purpose; for example, if it be desirable that a house should exclude the cold of winter, and the heat of summer, it certainly would not be advisable to form the roof, or any other considerable part of it, wholly of iron; as you

* While this Essay has been passing through the press, two accidents have occurred in London, and in one case a life was lost, besides a very considerable loss of property.

could not easily find another substance, for the purpose, that suffers heat to pass through it so rapidly as iron does. But, it is more imprudent to build heavy brick or stone walls upon timber supports, a material which is so subject to decay, and so easily destroyed by fire; and yet nearly half the houses in London are partly sustained by wooden posts. If you use timber to prevent settlements where a foundation is soft or irregular, the timber decays, and worse settlements take place than those it was intended to avoid; in all such cases iron might be used with success.

I think it will appear, on an accurate survey of the present state of the mechanical arts, that the physical and mechanical properties of materials are not sufficiently studied. If such knowledge were cultivated, if it formed a part of a young mechanic's education; that is, if he were prepared by a regular course of study respecting the nature and properties of materials, would not his progress in any particular art be greatly facilitated? Experience furnishes a practical mechanic with some share of this knowledge, but such experience is always limited to a particular range of objects, and it engenders prejudice in favour of particular things, and particular modes of operation.

Lord Bacon's idea of a Mechanical History*, which Diderot attempted to realize† is not so well calculated to fulfil his own views as a well-directed course of experiments on the nature, forms, and properties of materials, illustrated by a reference to the manner of applying them in the arts. In Chemistry much has been already done; but an experimental School of Mechanical Science remains to be formed.

Having briefly alluded to this deficiency in the experimental investigation of the mechanical properties of bodies, I must proceed to inform the reader of the nature of the work I now offer to his notice.

This work is divided into seven sections:— The *First Section* consists of introductory remarks on the use and qualities of cast iron, and of cautions to be observed in employing it. This section is followed by two extensive tables, which will often save the practical man a considerable share of trouble in calculation.

The *Second Section* explains the arrangement and use of the tables which precede it.

* Of the Advancement of Learning. Book II. Bacon's Works, Vol. i. p. 79.

† French Encyclopédie.

It is a common and a well understood fact, that an uniform beam is not equally strained in every part, and therefore may be reduced in size, so as to lessen both the strain and the expense of material.

The *Third Section* points out the value of cast iron, in this particular, and the forms to be adopted for different cases.

The *Fourth Section* contains a popular explanation of the strongest forms for the sections of beams; the construction of open beams; and the best form for shafts. A due consideration of these two sections will enable the young mechanic to guard against some common errors in attempting to apply these things to practice.

The *Fifth Section* is wholly devoted to experiments; it will be found to contain, in addition to my own experiments, almost all of the experiments on cast iron that have been described by preceding writers. Those I have tried for the purpose of establishing rules, to apply in practice, have been made with a different view of the subject from that entertained by preceding experimentalists; one better

adapted for practical application, one which shows that, within the proper limits, our theory of the strength of materials is to be depended upon; but that beyond these limits materials should never be strained in constructions of any kind whatever*. Nevertheless it would be extremely desirable that some accurate experiments on the extension of bodies should be made, when the strain exceeds the elastic force; as by that means something important regarding the ductility of matter might be discovered; and perhaps they might throw some light on the nature and arrangement of the ultimate particles of bodies.

In the *Sixth Section* I have shown how to obtain some of the most useful practical rules from the first principles that are furnished by experience. I have conducted the investigation of these rules in a manner somewhat different from other writers, and I have avoided the use of Fluxions†. Several new cases are investigated,

* To Dr. T. Young we are chiefly indebted for showing the necessity of attending to the strain which produces permanent alteration. Nat. Phil. vol. i. p. 141. To that valuable work I am most indebted for assistance in this Essay.

† I have rejected Fluxions in consequence of the very obscure manner in which its principles have been explained by the writers I

and some addition is made to the theory of resistance; the reader will find examples of this in treating of the strength of beams, art. 77 to 85; the deflexion of beams, art. 90 to 93; the strain upon beams, art. 96 to 104; the resistance to torsion, art. 222 to 227; and the resistance of columns, art. 230 to 246.

In the *Seventh Section* I have considered the resistance of beams to impulsive force. In this section will be found many important rules, with examples of their application to the moving parts of engines, bridges, &c.; wherein the advantage gained by employing beams of the figures of equal resistance is shown.

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have consulted on the subject. I cannot reconcile the idea of one of the terms of a proportion vanishing for the purpose of obtaining a correct result; it is not, it cannot be good reasoning; though, from other principles, I am aware that the conclusions obtained are correct. If the doctrine of Fluxions be freed from the obscure terms, limiting ratios, evanescent increments and decrements, &c. it is, in reality, not very difficult. If you represent the increase of a variable quantity by a progression, (as is done in art. 249. Sect. VII.) the first term of that progression is the same thing as what is called a fluxion; and the sum of the progression is the same as a fluent. A fluxion, is, therefore, the first increase of an increasing variable quantity, and the last decrease of a decreasing one; and the expansion of a variable quantity into a progression is the best and most clear comment that can be added to the Lemmas of Sir Isaac Newton.

The Seventh Section is followed by an extensive *Table of the Properties of Materials, and other Data, often used in Calculations*, arranged alphabetically. By means of this table the various rules for the strength of cast iron, contained in this work, may be applied to several other kinds of materials.

A Note, which I have added at the end of the table, on the chemical action of some bodies on cast iron, will be read with interest by those who employ cast iron where it is exposed to the action of sea water.

Each Plate is accompanied by a page of descriptive letter-press opposite to it, with references to the articles which the figures are intended to explain.

And, in general, it will be found that the Examples are selected with a view to explain the practical application of the rules; and to make the reader aware of the limits and precautions to be attended to. In fact the want of such information has often brought theory into discredit with some men, whereas the fault ought to have fallen on the person that misapplied it.

I hope there will be few things of any importance found in this work, for which a sufficient

reason is not given; sometimes I have been compelled to omit several steps in the investigations, in order to make it as little mathematical as possible; and such omissions the reader must excuse, till a larger share of mathematical learning becomes the common lot of every practical mechanic.

The communication of any experiment, or observation that is calculated to confirm or correct any thing I have done, I shall esteem a favour; for, should it meet with the encouragement I expect, it will soon be followed by a Second Part, on the Strength of Pipes, Mains, Tanks, Boilers, &c.; of Chains to resist Impulsion and Pressure; of Suspension Bridges; and of Framed Work.

16, *Grove Place, Lisson Grove,*
St. Mary-le-Bone, London,

March 16, 1822.

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All the quantities, proportions, &c. are stated in English weights and measures; the pound being the avoirdupois pound.

SECTION I.

Introduction.

Art I. In consequence of the security which cast iron gives, when it is properly employed, for supporting considerable weights, pressures, or moving forces, it has lately been very much used; and is likely to wholly supersede the use of timber for many important purposes. Indeed, so considerable are the improvements that have arisen out of its use, that the period of its general introduction has been very justly considered as forming a new era in the history of machines*. "All other improvements," it has been remarked, "have been limited; confined to particular machines; but this, having increased the strength and durability of every machine, has improved the whole†."

Cast iron is a valuable material, because it gives safety against fire, it is not liable to sudden decay,

* Essays on Mill-work, &c. by Robertson Buchanan, Essay II. p. 18.

† Mr. Dunlop's Account of some Experiments on Cast Iron. Dr. Thomson's Annals of Philosophy, vol. xiii. p. 200.

nor soon destroyed by wear and tear, and it can be easily moulded into the form of greatest strength, or that which is best adapted for our intended purpose.

The fatal consequences that might result from the use of timber for supporting heavy buildings, either in case of fire or of decay, have often been foreseen; but in a few instances it has happened, that where iron has been used for greater security against fire, the structure has failed from want of strength. Such failures have not occurred from any defect in the material itself; for it too often happens that such works are conducted by persons of little experience, and less scientific knowledge. People of this kind generally imagine that a large piece of iron is almost of infinite strength, and they have a like indistinct notion of pressure. The dimensions of the most important parts of structures are too often fixed by guess or chance; and the person who calculates the value of materials to the fraction of a penny, seldom if ever attempts to estimate their power, or the stress to which they will be exposed.

The manner in which the resistance of materials has been treated by most of our common mechanical writers, has also, in some degree, misled such practical men as were desirous of proceeding upon sure ground; and has given occasion for the sarcastic remark, " that the stability of a building is inversely proportional to the science of the builder*.

When it is considered that it is absolutely neces-

* Ency. Method. Dict. Architecture, art. Equilibre.

sary that the parts of a building or a machine should preserve a certain form or position, as well as that they should bear a certain stress, it will become obvious, that something more than the mere resistance to fracture should be calculated. In cases where the parts are short and bulky, it may do very well to employ the rules for resistance to fracture, and make the parts strong enough to sustain four times the load, but such cases rarely occur; and where long pieces are loaded to one-fourth of their strength, we may expect much flexure, vibration, and instability.

If a material of any kind be loaded with more than a certain quantity, it loses the power of recovering its natural form, when the load is removed; the arrangement of its particles undergoes a permanent alteration; and if it supports the same load during a considerable time, the deflexion will increase, and the more in proportion as the load is above the elastic force of the material.

On this part of the resistance of materials I have made many experiments, both with metals of various kinds, and with timber: I find, that while the elastic force, or power of restoration remains perfect, the extension is always directly proportional to the extending force, and that the deflexion does not increase after the load has been on for a second or two; but when the strain exceeds the elastic force, the extension or deflexion becomes irregular, and increases with time. I was led into this important inquiry by considering the proportions for cannon, and the common method of proving them. It ap-

pears from my experiments, that firing a certain number of times with the same quantity of powder, would burst a cannon when the strain is above the elastic force of the material, though the effect of the first charge might not be sensible.

In the moving parts of machines the strain should obviously be under the elastic force of the material, and in the second table will be found the flexure and load a piece of a given size will bear without destroying the elastic force.

I think every one who carefully examines the subject, will feel satisfied that the measure of the resistance of a material to flexure, is the only proper measure of its resistance when it is to be applied in the construction of buildings, and that of its resistance to permanent alteration when it is used for machines.

In order to supply practical men with a convenient and ready means of assigning the dimensions of cast-iron beams to support known pressures, or moving forces, I have drawn up this small tract. I am persuaded that its portability and usefulness will find it a place among the common works of reference, which are more or less necessary to every architect, engineer, and builder. To bring it within as small a compass as possible, I have arranged the tables so as to include as many distinct applications as the nature of the subjects seemed capable of admitting.

Some Particulars to be observed in using the Tables.

2. The weight of the beam itself is always to be estimated, and added to the load to be supported;

or (because this method renders it necessary to estimate the weight before the bulk be determined) find the dimensions of the piece that would support the load by one of the tables, and increase the breadth in the same proportion as the weight of the piece increases the load. If the weight of the piece, for example, be an eighth part of the load, then to the breadth, found by the table, add an eighth part of that breadth; and so of any other proportion. It is not an absolutely correct method, but it is simple and correct enough for use.

3. The tables and rules are calculated for soft gray cast iron. Metal of this kind yields easily to the file when the external crust is removed, and is slightly malleable in a cold state. Dr. C. Hutton has justly given the preference to such iron, because it is "less liable to fracture by a blow, or shock, than the hard metal*."

White cast iron is less subject to be destroyed by rusting than the gray kind; and it is also less soluble in acids; therefore it may be usefully employed where hardness is necessary, and where its brittleness is not a defect; but it should not be chosen for bearing purposes.

White cast iron, in a recent fracture, has a white and radiated appearance, indicating a crystalline structure. It is very brittle and hard.

Gray cast iron has a dull granulated fracture, of a

* Tracts, vol. i. p. 141.

gray colour; it is much softer and tougher than the white cast iron.

But between these kinds there are varieties of cast-iron, having various shades of these qualities; those should be esteemed the best which approach nearest to the gray cast iron.

Gray cast iron is used for artillery, and is sometimes called gun-metal.

The best and most certain test of the quality of a piece of cast iron, is to try the edge with a hammer; if the blow of a hammer make a slight impression, denoting some degree of malleability, the iron is of a good quality, provided it be uniform; if fragments fly off, and no sensible indentation be made, the iron will be hard and brittle.

It has been remarked that "iron varies in strength, and not only from different furnaces, but also from the same furnace and the same melting; but this seems to be owing to some imperfection in the casting, and in general iron is much more uniform than wood*." I am glad to find my own experience supported by the opinion of a writer so well known to practical men as Mr. Banks. Cast iron when it fails gives no warning of its approaching fracture, which is the chief defect it has when employed to sustain weights; therefore care should be taken to give it sufficient strength.

4. The parts of each casting should be kept as

* Banks on the Power of Machines, p. 73. See also p. 94. of the same work.

nearly of the same bulk as possible, in order that they may all cool at the same rate.

Great care should be taken to prevent air bubbles in castings; and the more time there can be allowed for cooling the better, because the iron will be tougher than when rapidly cooled; slow cooling answers the same purpose as annealing.

In making patterns for cast iron, an allowance of about one-eighth of an inch per foot, must be made for the contraction of the metal in cooling. Also the patterns that require it should be slightly bevelled to allow of their being drawn out of the sand without injuring the impression.

TABLE I.—ART. 5. *A Table of the Depths of Square Beams or one Cwt. to 500 Tons, when supported at the ends, and loaded in in length.*

Lengths in feet.		4	6	8	10	12	14	16	18	20
Weight in tons.	Weight in pounds.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.
$\frac{1}{20}$	112	1·2	1·4	1·7	1·9	2·0	2·2	2·4	2·5	2·6
$\frac{1}{10}$	224	1·4	1·7	2·0	2·2	2·4	2·6	2·8	3·0	3·1
$\frac{3}{20}$	336	1·6	1·9	2·2	2·4	2·7	2·9	3·1	3·3	3·4
$\frac{1}{5}$	448	1·7	2·0	2·4	2·6	2·9	3·1	3·3	3·5	3·7
$\frac{1}{4}$	560	1·8	2·2	2·5	2·8	3·0	3·3	3·5	3·7	3·9
$\frac{3}{10}$	672	1·8	2·2	2·6	2·9	3·2	3·4	3·7	3·9	4·1
$\frac{7}{20}$	784	1·9	2·3	2·7	3·0	3·3	3·6	3·8	4·1	4·2
$\frac{2}{5}$	896	2·0	2·4	2·8	3·1	3·4	3·7	3·9	4·2	4·4
$\frac{9}{20}$	1008	2·0	2·5	2·9	3·2	3·5	3·8	4·0	4·3	4·5
$\frac{1}{2}$	1120	2·1	2·6	3·0	3·3	3·6	3·9	4·2	4·4	4·7
$\frac{11}{20}$	1232	2·1	2·6	3·0	3·4	3·7	4·0	4·3	4·5	4·8
$\frac{3}{5}$	1344	2·2	2·7	3·1	3·5	3·8	4·1	4·4	4·7	4·9
$\frac{13}{20}$	1456	2·2	2·7	3·1	3·5	3·8	4·2	4·4	4·7	4·9
$\frac{7}{10}$	1568	2·3	2·8	3·2	3·6	3·9	4·2	4·5	4·8	5·0
$\frac{3}{4}$	1680	2·3	2·8	3·2	3·6	4·0	4·3	4·6	4·9	5·2
$\frac{4}{5}$	1792	2·4	2·9	3·3	3·7	4·0	4·4	4·7	5·0	5·2
$\frac{17}{20}$	1904	2·4	2·9	3·4	3·8	4·1	4·4	4·7	5·0	5·3
$\frac{9}{10}$	2016	2·4	3·0	3·4	3·8	4·2	4·5	4·8	5·1	5·4
$\frac{19}{20}$	2128	2·5	3·0	3·5	3·9	4·2	4·6	4·9	5·2	5·4
1	2240	2·5	3·0	3·5	3·9	4·3	4·6	4·9	5·2	5·5
$1\frac{1}{4}$	2800	2·6	3·2	3·7	4·1	4·5	4·9	5·2	5·5	5·8
Deflex. in inches.		·1	·15	·2	·25	·3	·35	·4	·45	·5

Bars of Cast Iron, of different lengths, to sustain weights of from the middle; the deflexion not to exceed $\frac{1}{40}$ of an inch for each foot*

22	24	26	28	30	32	34	36	38	40	
Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Weight.
2·7	2·9	3·0	3·1	3·2	3·3	3·4	3·5	3·6	3·7	1 cwt.
3·3	3·4	3·6	3·7	3·8	3·9	4·1	4·2	4·3	4·4	2 —
3·6	3·8	3·9	4·1	4·2	4·3	4·5	4·6	4·7	4·8	3 —
3·9	4·0	4·2	4·3	4·5	4·7	4·8	4·9	5·0	5·2	4 —
4·1	4·3	4·4	4·6	4·8	4·9	5·1	5·2	5·4	5·5	5 —
4·3	4·5	4·6	4·8	5·0	5·1	5·3	5·4	5·6	5·8	6 —
4·4	4·6	4·8	5·0	5·2	5·4	5·5	5.7	5·9	6·0	7 —
4·6	4·8	5·0	5·2	5·4	5·6	5·7	5·9	6·0	6·2	8 —
4·7	4·9	5·1	5·3	5·5	5·7	5·9	6·0	6·2	6·4	9 —
4·9	5·2	5·3	5·4	5·7	5·9	6·0	6·2	6·4	6·5	10 —
5·0	5·3	5·4	5·6	5·8	6·0	6·2	6·4	6·5	6·7	11 —
5·1	5·3	5·5	5·7	5·9	6·1	6·3	6·5	6·7	6·8	12 —
5·2	5·4	5·6	5·9	6·0	6·2	6·5	6·6	6·8	7·0	13 —
5·3	5·5	5·7	6·0	6·1	6·4	6·6	6·7	6·9	7·1	14 —
5·4	5·6	5·8	6·1	6·2	6·5	6·7	6·8	7·0	7·2	15 —
5·5	5·7	5·9	6·2	6·4	6·6	6·8	6·9	7·2	7·4	16 —
5·5	5·8	6·0	6·2	6·5	6·7	6·9	7·1	7·3	7·5	17 —
5·6	5·9	6·1	6·4	6·6	6·8	7·0	7·2	7·4	7·6	18 —
5·7	6·0	6·2	6·5	6·7	6·9	7·1	7·3	7·5	7·7	19 —
5·8	6·0	6·3	6·5	6·8	7·0	7·2	7·4	7·5	7·8	1 ton.
6·1	6·4	6·6	6·9	7·2	7·4	7·6	7·8	8·0	8·2	1½ —
·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0	Defl. In.

* The weight of the load to be supported must include the weight of the beam.

TABLE I.

Lengths in feet.		4	6	8	10	12	14	16	18	20
Weight in tons.	Weight in pounds.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.
1½	3,360	2·8	3·4	3·9	4·3	4·7	5·1	5·5	5·8	6·1
1¾	3,920	2·9	3·5	4·0	4·5	4·9	5·3	5·7	6·0	6·3
2	4,480	2·9	3·5	4·1	4·7	5·1	5·5	5·9	6·2	6·5
2½	5,600	3·1	3·8	4·4	4·9	5·5	5·8	6·2	6·6	6·9
3	6,720	3·3	4·0	4·6	5·1	5·7	6·1	6·5	6·9	7·3
3½	7,840	3·4	4·1	4·8	5·3	5·8	6·3	6·7	7·1	7·5
4	8,960	3·5	4·3	4·9	5·5	6·0	6·5	7·0	7·4	7·8
4½	10,080	3·6	4·4	5·1	5·7	6·2	6·7	7·2	7·6	8·0
5	11,200	3·7	4·5	5·2	5·8	6·4	6·9	7·4	7·8	8·2
6	13,440	3·9	4·7	5·5	6·1	6·7	7·2	7·7	8·2	8·6
7	15,680	4·0	4·9	5·7	6·3	6·9	7·5	8·0	8·5	8·9
8	17,920		5·1	5·9	6·6	7·2	7·8	8·3	8·8	9·3
9	20,160		5·2	6·0	6·8	7·4	8·0	8·5	9·0	9·5
10	22,400		5·3	6·2	6·9	7·6	8·2	8·8	9·3	9·8
11	24,640		5·5	6·4	7·1	7·8	8·4	9·0	9·5	10·0
12	26,880		5·6	6·5	7·2	7·9	8·6	9·2	9·7	10·2
13	29,120		5·7	6·6	7.4	8·1	8·8	9·4	9·9	10·4
14	31,360		5·8	6·8	7·5	8·3	8·9	9·5	10·1	10·6
15	33,600		6·0	6·9	7·7	8·4	9·1	9·7	10·3	10·8
16	35,840			7·0	7·8	8·5	9·2	9·8	10·4	11·0
17	38,080			7·1	7·9	8·7	9·4	10·0	10·6	11·2
18	40,320			7·2	8·0	8·8	9·5	10·1	10·8	11·3
19	42,560			7·3	8·1	8·9	9·6	10·3	10.9	11·5
Deflex. in inches.		·1	·15	·2	·25	·3	·35	·4	·45	·5

Continued.

22	24	26	28	30	32	34	36	38	40	
Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Weight in Tons.
6·4	6·7	7·0	7·2	7·5	7·7	8·0	8·2	8·4	8·6	1½
6·7	6·9	7·2	7·5	7·7	8·0	8·2	8·5	8·7	8·9	1¾
6·8	7·2	7·6	7·7	8·0	8·3	8·5	8·7	9·0	9·2	2
7·3	7·6	7·9	8·2	8·5	8·8	9·0	9·3	9·6	9·8	2½
7·6	7·9	8·3	8·6	8·9	9·2	9·4	9·7	10·0	10·1	3
7·9	8·2	8·6	8·9	9·2	9·5	9·8	10·1	10·4	10·6	3½
8·2	8·5	8·9	9·2	9·5	9·8	10·1	10·4	10·7	11·0	4
8·4	8·8	9·1	9·5	9·8	10·1	10·4	10·8	11·0	11·4	4½
8·6	9·0	9·4	9·7	10·1	10·4	10·7	11·0	11·2	11·6	5
9·0	9·4	9·8	10·2	10·5	10·9	11·2	11·5	11·9	12·1	6
9·4	9·8	10·2	10·6	11·0	11·3	11·7	12·0	12·3	12·7	7
9·7	10·1	10·6	10·9	11·3	11·7	12·0	12·4	12·8	13·1	8
10·0	10·4	10·9	11·3	11·7	12·0	12·4	12·8	13·1	13·5	9
10·3	10·7	11·2	11·6	12·0	12·4	12·8	13·1	13·5	13·8	10
10·5	11·0	11·5	11·9	12·3	12·7	13·1	13·5	13·8	14·2	11
10·8	11·2	11·7	12·1	12·5	13·0	13·4	13·7	14·1	14·5	12
11·0	11·5	11·9	12·4	12·8	13·2	13·6	14·0	14·4	14·7	13
11·1	11·7	12·1	12·6	13·0	13·4	13·8	14·2	14·6	15·0	14
11·4	11·9	12·3	12·8	13·2	13·7	14·1	14·5	14·9	15·3	15
11·5	12·0	12·5	13·0	13·5	13·9	14·3	14·7	15·1	15·5	16
11·7	12·2	12·7	13·2	13·7	14·1	14·5	14·9	15·4	15·8	17
11·9	12·4	12·9	13·4	13·9	14·3	14·7	15·1	15·6	16·0	18
12·0	12·6	13·1	13·6	14·1	14·5	15·0	15·4	15·8	16·2	19
·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0	Deflex.

TABLE I.

Lengths in ft.		4	6	8	10	12	14	16	18	20
Wt. in Tons.	Weight in lbs.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.
20	44,800			7·4	8·2	9·0	9·7	10·4	11·0	11·6
22	49,280			7·5	8·4	9·2	10·0	10·7	11·3	11·9
24	53,760			7·7	8·6	9·4	10·2	10·9	11·5	12·2
26	58,240			7·9	8·8	9·6	10·4	11·1	11·8	12·4
28	62,720			8·0	8·9	9·8	10·6	11·4	12·0	12·7
30	67,200				9·1	10·0	10·8	11·5	12·2	12·9
32	71,680				9·3	10·2	11·0	11·7	12·4	13·1
34	76,160				9·4	10·3	11·1	11·9	12·6	13·3
36	80,640				9·5	10·4	11·3	12·0	12·8	13·4
38	85,120				9·7	10·6	11·4	12·2	13·0	13·6
40	89,600				9·8	10·7	11·6	12·4	13·1	13·8
42	94,080				9·9	10·9	11·7	12·5	13·3	14·0
44	98,560				10·0	11·0	11·9	12·7	13·5	14·2
46	103,040					11·1	12·0	12·8	13·6	14·3
48	107,520					11·2	12·1	13·0	13·7	14·5
50	112,000					11·3	12·2	13·1	13·9	14·6
52	116,480					11·5	12·4	13·2	14·0	14·7
54	120,960					11·6	12·5	13·3	14.1	14·9
56	125,440					11·7	12·6	13·5	14·3	15·0
58	129,920					11·8	12·7	13·6	14·4	15·1
60	134,400					11·9	12;8	13·7	14·5	15·3
65	145,600						13·1	14·0	14·8	15·6
70	156,800						13·3	14·3	15·1	15·9
Deflex. in In.		·1	·15	·2	·25	·3	·35	·4	·45	·5

Continued.

22	24	26	28	30	32	34	36	38	40	
Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Depth Inches.	Wt. in Tons.
12·2	12·7	13·2	13·8	14·2	14·7	15·1	15·6	16·0	16·4	20
12·5	13·0	13·6	14·1	14·6	15·1	15·5	15·9	16·4	16·8	22
12·8	13·4	13·9	14·4	14·9	15·4	15·9	16·3	16·8	17·2	24
13·0	13·6	14·2	14·7	15·2	15·7	16.2	16·7	17·1	17·6	26
13·3	13·9	14·4	15·0	15·5	16·0	16·5	17·0	17·4	17·9	28
13·5	14·1	14·7	15·2	15·7	16·3	16·8	17·3	17·7	18·2	30
13·7	14·3	14·9	15·5	16·0	16·5	17·0	17·5	18·0	18·5	32
13·9	14·5	15·1	15·7	16·2	16·8	17·3	17·8	18·3	18·8	34
14·1	14·7	15·3	15·9	16·5	17·0	17·5	18.0	18·5	19·0	36
14·3	14·9	15·5	16·1	16·7	17·2	17·8	18·3	18·8	19·3	38
14·5	15·1	15·7	16·4	16·9	17·5	18·0	18·5	19·1	19·5	40
14·7	15·3	15·9	16·5	17·1	17·7	18·2	18·7	19·3	19·8	42
14·9	15·5	16·1	16·8	17·4	17·9	18·5	19·0	19·5	20·0	44
15·0	15·7	16·3	17·0	17·6	18·1	18·7	19·2	19·8	20·3	46
15·2	15·9	16·5	17·1	17·7	18·3	18·8	19·4	20·0	20·5	48
15·3	16·0	16·6	17·3	17·9	18·5	19·0	19·6	20·1	20·7	50
15·5	16·2	16·8	17·5	18·1	18·7	19·2	19·8	20·3	21·0	52
15·6	16·3	17·0	17·6	18·2	18·8	19·4	19·9	20·5	21·1	54
15·8	16·5	17·1	17·8	18·4	19.0	19·6	20·1	20·7	21·3	56
15·9	16·6	17·3	17·9	18·5	19·2	19·7	20·3	20·9	21·4	58
16·0	16·7	17·4	18·1	18·7	19·3	19·9	20·5	21·1	21·6	60
16·4	17·1	17·8	18·5	19·1	19·8	20·4	20·9	21·5	22·1	65
16·7	17·4	18·2	18·8	19.5	20·1	20·8	21·3	22·0	22·5	70
·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0	Defl.

TABLE I.

Lengths in feet.		4	6	8	10	12	14	16	18	20
Weight in Tons.	Weight in lbs.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.
75	168,000						13·6	14·5	15·4	16·2
80	179,200						13·8	14·7	15·6	16·4
85	190,400						14·0	14·9	15·8	16·7
90	201,600							15·2	16·1	16·9
95	212,800							15·4	16·3	17·2
100	224,000							15·6	16·5	17·4
110	246,400							15·9	16·8	17·8
120	268,800								17·2	18·2
130	291,200								17·7	18·6
140	313,600								17·9	19·0
150	336,000									19·3
160	358,400									19·6
170	380,800									19·9
180	403,200									
190	425,600									
200	448,000									
250	560,000									
300	672,000									
350	784,000									
400	896,000									
450	1,008,000									
500	1,120,000									
Deflex. in inches.		·1	·15	·2	·25	·3	·35	·4	·45	·5

Continued.

22	24	26	28	30	32	34	36	38	40	
Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Weight in Tons.
17·0	17·7	18·5	19·2	19·8	20·5	21·3	21·7	22·3	22·9	75
17·2	18·0	18·7	19·4	20·1	20·7	21·4	22·0	22·6	23·2	80
17·5	18·3	19·0	19·7	20·4	21·0	21·7	22·4	23·0	23·6	85
17·8	18·6	19·3	20·0	20·7	21·4	22·1	22·7	23·3	23·9	90
18·0	18·8	19·5	20·3	21·0	21·7	22·4	23·0	23·6	24·3	95
18·2	19·0	19·8	20·6	21·3	22·0	22·6	23·3	23·9	24·5	100
18·7	19·5	20·3	21·0	21·8	22·5	23·2	23·8	24·5	25·1	110
19·1	19·9	20·8	21·5	22·3	23·0	23·7	24·4	25·0	25·7	120
19·5	20·4	21·3	22·0	22·7	23·5	24·2	24·9	25·5	26·3	130
19·9	20·8	21·7	22·5	23·2	23·9	24·6	25·4	26·0	26·7	140
20·2	21·1	22·0	22·8	23·6	24·3	25·0	25·8	26·5	27·2	150
20·5	21·5	22·3	23·1	23·9	24·7	25·5	26·2	26·9	27·6	160
20·8	21·8	22·6	23·5	24·3	25·1	25·9	26·6	27·4	28·1	170
21·1	22·1	23·0	23·9	24·7	25·5	26·3	27·0	27·8	28·5	180
21·5	22·4	23·3	24·2	25·0	25·9	26·7	27·4	28·2	28·9	190
21·7	22·7	23·6	24·5	25·3	26·2	27·0	27·7	28·5	29·2	200
	23·9	24·9	25·9	26·8	27·6	28·5	29·4	30·1	31·0	250
		26·0	27·1	28·0	28·9	29·9	30·7	31·5	32·0	300
			28·0	29·0	30·0	31·0	32·0	33·0	33·7	350
				30·0	31·1	32·0	32·9	33·9	34·7	400
					32·0	33·1	34·0	34·8	35·8	450
						33·8	34·8	35·7	36·7	500
·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0	Deflex.

TABLE II.—ART. 6. *A Table shewing the weight or pressure a ing its elastic force, when it is supported at the ends, and loaded which that weight will produce.*

Lengths.	1 foot.		2 feet.		3 feet.		4 feet.		5 feet.	
Depths.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches	Weight in lbs.	Defl. in inches.
1 in.	850	·02	425	·08	283	·18	212	·32	170	·5
1½—	1,912	·014	956	·053	637	·12	477	·21	383	·33
2 —			1,700	·04	1,132	·09	848	·16	680	·25
2½—			2,656	·032	1,769	·072	1,325	·128	1,062	·2
3 —					2,547	·06	1,908	·11	1,530	·167
3½—					3,467	·052	2,597	·092	2,082	·143
4 —							3,392	·08	2,720	·125
4½—							4,293	·071	3,442	·111
5 —									4,250	·1
6 —									6,120	·083
7 —										
8 —										
9 —										
10 —										
11 —										
12 —										
13 —										
14 —										

REMARK.—The load shewn by this table is the greatest a beam should ever sustain, and, therefore, in calculating this load, ample allowance must be made for accidents, and the weight of the beam itself must be included.

beam of cast-iron, one inch in breadth, will sustain without destroy-in the middle of its length, and also the deflexion in the middle

6 feet.		7 feet.		8 feet.		9 feet.		10 feet.		Lengths.
Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in In.	Weight in lbs.	Defl. in In.	Weight in lbs.	Defl. in In.	Depths.
142	·72	121	·98	106	1·28	95	1·62	85	2·0	1 in.
320	·48	273	·65	239	·85	214	1·08	192	1·34	1½—
568	·36	484	·49	425	·64	380	·81	340	1·0	2 —
887	·29	756	·39	662	·51	594	·65	531	·8	2½—
1,278	·24	1,089	·33	954	·426	855	·54	765	·66	3 —
1,739	·205	1,482	·28	1,298	·365	1,164	·46	1,041	·57	3½—
2,272	·18	1,936	·245	1,700	·32	1,520	·405	1,360	·5	4 —
2,875	·16	2,450	·217	2,146	·284	1,924	·36	1,721	·443	4½—
3,560	·144	3,050	·196	2,650	·256	2,375	·32	2,125	·4	5 —
5,112	·12	4,356	·163	3,816	·213	3,420	·27	3,060	·33	6 —
6,958	·103	5,929	·14	5,194	·183	4,655	·23	4,165	·29	7 —
9,088	·09	7,744	·123	6,784	·16	6,080	·203	5,440	·25	8 —
		9,801	·109	8,586	·142	7,695	·18	6,885	·22	9 —
		12,100	·098	10,600	·128	9,500	·162	8,500	·2	10 —
				12,826	·117	11,495	·15	10,285	·182	11 —
				15,264	·107	13,680	·135	12,240	·17	12 —
						16,100	·125	14,400	·154	13 —
						18,600	·115	16,700	·143	14 —

TABLE II.

Lengths.	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.	
Depths.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Defl. in In.
2in.	283	1·44	243	1·96	212	2·56	189	3·24	170	4·0
3—	637	·96	546	1·31	478	1·71	425	2·16	382	2·67
4—	1,133	·72	971	·98	849	1·28	755	1·62	680	2·08
5—	1,771	·58	1,518	·78	1,328	1·02	1,180	1·29	1,062	1·6
6—	2,548	·48	2,184	·65	1,912	·85	1,699	1·08	1,530	1·34
7—	3,471	·41	2,975	·58	2,603	·73	2,314	·93	2,082	1·14
8—	4,532	·36	3,884	·49	3,396	·64	3,020	·81	2,720	1·00
9—	5,733	·32	4,914	·44	4,302	·57	3,825	·72	3,438	·89
10—	7,083	·288	6,071	·392	5,312	·512	4,722	·648	4,250	·8
11—	8,570	·26	7,346	·36	6,426	·47	5,714	·59	5,142	·73
12—	10,192	·24	8,736	·33	7,648	·43	6,796	·54	6,120	·67
13—	11,971	·22	10,260	·307	8,978	·39	7,980	·49	7,182	·61
14—	13,883	·21	11,900	·28	10,412	·36	9,255	·46	8,330	·57

Continued.

22 feet.		24 feet.		26 feet.		28 feet.		30 feet.		Lengths.
Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Weight in lbs.	Deflex. in In.	Depths.
154	4·84	142	5·76	131	6·76	121	7·84	113	9·0	2 in.
347	3·23	318	3·84	294	4·51	273	5·23	255	6·0	3—
618	2·42	566	2·88	523	3·38	485	3·92	453	4·5	4—
966	1·93	885	2·30	817	2·70	759	3·14	708	3·6	5—
1,390	1·61	1,274	1·92	1,176	2·25	1,092	2·61	1,019	3·0	6—
1,893	1·38	1,735	1·65	1,602	1·93	1,487	2·24	1,388	2.57	7—
2,472	1·21	2,264	1·44	2,092	1·69	1,940	1·96	1,812	2·25	8—
3,123	1·07	2,862	1·28	2,646	1·50	2,457	1·74	2,295	2·0	9—
3,863	·968	3,541	1·152	3,269	1·352	3,035	1·568	2,833	1·8	10—
4,675	·88	4,285	1·05	3,955	1·23	3,673	1·425	3,428	1·64	11—
5,560	·81	5,096	·96	4,704	1·13	4,368	1·31	4,076	1·5	12—
6,529	·74	5,985	·886	5,525	1·04	5,130	1·21	4,788	1·38	13—
7,573	·69	6,941	·824	6,408	·965	5,950	1·12	5,553	1·28	14—

TABLE II.

Length	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.	
Depths.	Weight in lbs.	Defl. in inches	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.
15 in.	15,937	·19	13,660	·26	11,952	·34	10,624	·43	9,562	·533
16—	18,128	·18	15,536	·245	13,584	·32	12,080	·403	10,880	·5
17—	20,500	·17	17,500	·23	15,353	·3	13,647	·38	12,282	·47
18—	22,932	·16	19,656	·217	17,208	·284	15,700	·36	13,752	·442
19—	25,404	·152	21,800	·207	19,053	·27	16,935	·34	15,242	·42
20—	28,332	·144	24,284	·196	21,248	·256	18,888	·324	17,000	·4
21—	31,230	·138	26,770	·186	23,426	·245	20,825	·31	18,742	·382
22—	34,500	·131	29,300	·178	25,712	·235	22,855	·295	20,570	·365
23—	37,600	·127	32,000	·17	28,103	·225	24,980	·282	22,482	·35
24—	40,768	·12	34,944	·163	30,592	·216	27,184	·27	24,480	·335
25—			37,700	·156	33,203	·21	29,514	·26	26,562	·32
26—			40,900	·15	35,912	·197	31,922	·25	28,730	·307
27—			44,000	·143	38,728	·19	34,425	·24	30,982	·297
28—			47,300	·14	41,650	·183	37,022	·28	33,320	·286
29—					44,678	·176	39,714	·223	35,742	·275
30—					47,808	·170	42,498	·216	38,250	·266
31—					51,053	·164	45,380	·207	40,842	·257
32—					54,400	·16	48,371	·202	43,520	·25
33—							51,425	·196	46,282	·242
34—							54,586	·19	49,130	·235
35—							57,847	·185	52,062	·228
36—							61,200	·18	55.080	·222

Continued.

22 feet.		24 feet.		26 feet.		28 feet.		30 feet.		Lengths.
Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Depths.
8,692	·645	7,967	·75	7,355	·9	6,829	1·03	6,374	1·2	15 in.
9,888	·63	9,056	·72	8,368	·84	7,760	·98	7,248	1·13	16—
11,166	·567	10,235	·673	9,447	·79	8,773	·92	8,188	1·06	17—
12,492	·54	11,448	·64	10,584	·75	9,828	·87	9,180	1·0	18—
13,857	·51	12,702	·607	11,725	·71	10,887	·825	10,161	·95	19—
15,452	·484	14,164	·576	13,076	·676	12,140	·784	11,332	·9	20—
17,036	·45	15,618	·55	14,417	·645	13,387	·75	12,495	·86	21—
18,700	·44	17,141	·525	15,823	·615	14,693	·71	13,713	·815	22—
20,439	·42	18,735	·5	17,286	·59	16,059	·68	14,988	·78	23—
22,240	·402	20,384	·48	18,816	·565	17,492	·665	16,304	·75	24—
24,148	·387	22,135	·46	20,432	·54	18,973	·625	17,703	·72	25—
26,118	·375	23,941	·443	22,100	·52	20,521	·607	19,153	·695	26—
28,166	·36	25,819	·427	23,832	·5	22,130	.58	20,655	·667	27—
30,290	·347	27,766	·41	25,630	·48	23,800	·56	22,213	·645	28—
32,493	·333	29,785	·395	27,494	·462	25,530	·54	23,828	·62	29—
34,767	·322	31,869	·384	29,421	·450	27,315	·522	25,497	·60	30—
37,148	·31	34,035	·37	31,417	·435	29,173	·505	27,228	·58	31—
39,563	·302	36,266	·36	33,477	·42	31,086	·49	29,013	·56	32—
42,075	·293	38,568	·35	35,602	·41	33,058	·47	30,855	·545	33—
44,663	·283	40,941	·336	37,792	·395	35,093	·46	32,753	·53	34—
47,329	·276	43,385	·329	40.048	·386	37,187	·448	34,708	·514	35—
50,073	·269	45,900	·32	42,369	·375	39,343	·435	36,720	·5	36—

SECTION II.

EXPLANATION OF THE TABLES, WITH EXAMPLES OF THEIR USE.

Explanation of the first Table.

7. The first table (page 8. art. 5.) shows by inspection, the dimensions of square beams to sustain weights or pressures of from one hundred weight to 500 tons; so as not be bent or deflected in the middle, more than one-fortieth of an inch for each foot in length.

The length is the distance between the supports, as AB, *Fig.* 1, *Plate* I. and the stress, whether it be from weight or pressure, is supposed to act at the middle of the length, as at C in the figure. The breadth and depth are supposed to be the same in every part of the length, and equal to one another.

The horizontal row of figures at the top of the table, contains the lengths in feet.

The columns at the outsides, contain the weights in cwts. and tons, and the second column on the left-hand side, contains the weights in pounds avoirdupois

The horizontal row of figures at the bottom shows the deflexion for each length. The other columns show the depths in inches.

Explanation of the second Table.

8. The second table (page 16, art 6.) is intended to show the greatest weight a beam of cast iron will bear in the middle of its length, when it is loaded with as much as it will bear, so as to recover its natural form when the load is removed. If a beam be loaded beyond that point, the equilibrium of its parts is destroyed, and it takes a permanent set. Also, in a beam so loaded beyond its strength, the deflexion becomes irregular, increasing very rapidly in proportion to the load.

The horizontal row of figures along the top of the table, contains the lengths in feet, that is, the distance between the points of support.

The columns on the outsides contain the depths in inches.

The other columns contain the weights in pounds avoirdupois, and the deflexions they would produce in inches and decimal parts, when the beams will be only just capable of restoring themselves.

The breadth of each beam is one inch, therefore the table shows the utmost weight a beam of one inch in breadth should have to bear, and a piece five inches in breadth will bear five times as much, and so of any other breadth.

Examples and Use of the Tables.

9. Example I. To find the depth of a square bar

of cast iron, twenty feet in length, that would support ten tons, the deflexion not exceeding half an inch.

Find the column in Table I. which has the length twenty feet at the top, and in that column, and opposite to ten tons in either of the side columns, will be found the proper depth for the bar, which is 9·8 inches.

10. Example II. Required the weight a beam would support without impairing its elastic force, the length, breadth, and depth being given?

Let the length be twenty feet and the breadth the same as the depth, ten inches. In the second table under the length twenty feet, and opposite the depth ten inches, we find the weight 4,250 lbs. for the load a beam one inch in breadth would bear, and this multiplied by 10, gives 42,500 lbs. or nearly nineteen tons; and the deflexion would be 0·8 inches, but the weight of the beam itself would be nearly three tons, and its effect the same as if half the three tons were applied in the middle, consequently the greatest load that the beam should be liable to sustain should not exceed seventeen tons and a half.

11. There are cases where a greater degree of flexure may be allowed, and there are others where it ought to be less; but I consider that to which the first table is calculated as nearly the mean, and it is easy to make any variation in this respect.

Example III. Let it be required to find the depth of a square cast iron bar to support ten tons without more deflexion than one-tenth of an inch, the length being twenty feet.

By examining the deflexion for twenty feet at the foot of the column in Table I. it will be found five times one-tenth of an inch, hence take the depth opposite five times the weight or fifty tons, which is 14·6 inches, the depth required.

12. Example IV. Find the depth of a square bar of cast-iron to support ten tons, the deflexion not to exceed one inch, the length being twenty feet.

This degree of deflexion is double that at the foot of the column headed twenty feet in Table I. therefore look opposite half the weight, or five tons, and the depth will be found to be 8·2 inches.

I have taken the same length and weight in each of these examples for the purpose of showing how much the depth must be increased to give stiffness.

13. When a bar or beam is employed to support a load in the middle, or at any other point of the length, a great saving of the material is made by making the bar thin and deep*, provided it be not made so thin as to break sideways.

The depth of a beam is sometimes limited by circumstances, and as no proportion could be given that would suit for every purpose, it is left entirely to the judgment of the person who may use the table. But there is a limit to the depth which if it be exceeded, renders the use of cast iron for bearing purposes very objectionable and dangerous, for if the depth be increased, it renders a beam rigid or nearly

* The term depth is always employed for the dimension in the direction of the pressure.

inflexible, and then a comparatively small impulsive force will break it. A very rigid beam resembles a hard body, it will bear an immense pressure, but the stroke of a small hammer will fracture it.

In order to mark the point where the depth has arrived at that proportion of the length which makes it become dangerously rigid, I have stopped the column of depths at that point, and should it be required to sustain a greater weight, the breadth must be increased instead of the depth.

14. Example v. Find the depth of a rectangular bar of cast iron to support a weight of ten tons in the middle of its length, the deflexion not to exceed one-fortieth of an inch per foot in length, and the length twenty feet; also let the depth be six times the breadth.

Under the length twenty feet in Table I. and opposite six times the weight, will be found the depth which in this case is 15·3 inches, and the breadth will be one-sixth of this depth, or 2·6 inches.

In the same manner, if the depth had been fixed to be four times the breadth, look opposite four times the weight for the depth, and make the breadth one-fourth of the depth, and so of any other proportion.

15. Example vi. If the breadth and length of a beam be given, and it be required to find the depth such that the beam may sustain a given weight without impairing its elastic force; then, in the second table, the depth and deflexion may be found thus: Divide the given weight by the breadth; the quo-

tient will be the weight a beam of one inch in breadth would sustain, which being found in the column of weights under the given length, the depth required will be opposite to it, and also the deflexion.

Let the given breadth be three inches, the weight to be supported ten tons or 22,400 lbs, and the length twenty feet. Then $\frac{22400}{3} = 7{,}466$; and the weight nearest to 7,466 lbs. in the column for twenty feet lengths in the second table is 8,330, and the depth fourteen inches, and the deflexion would be 0·57 inches.

16. Example VII. To find the diameter of a solid cylinder of cast iron, that will bear a given pressure, the flexure in the middle not to exceed one-fortieth of an inch for each foot in length.

Let us suppose the distance of the supported points of a shaft to be twenty feet, and the pressure to be equal to ten tons. Then multiply the pressure by the constant multiplier 1·7 *, that is, $10 \times 1{\cdot}7 = 17$, and in this case, opposite seventeen tons in the first table, and under twenty feet, we find 11·2 inches for the diameter of the cylinder or shaft.

17. Example VIII. When the diameter of a solid cylinder is given, and the length, to find the greatest load it will sustain without injury to its elasticity, and the deflexion that weight will cause.

Suppose the diameter to be eleven inches, and the length twenty feet, then in the second table opposite

* See Elementary Principles of Carpentry, Sect. II. art. 96.

the depth eleven inches, and under the length twenty feet, will be found 5,142 lbs. Let this be multiplied by the diameter eleven inches, and divided by the constant number 1·7; the result will be the weight required in lbs.

In this case it is 33,271 lbs. for $5142 \times 11 \div 1{\cdot}7 = 33{,}271$. The deflexion opposite eleven inches and under twenty feet, is ·73 inches.

Any different degree of deflexion may be allowed for in the same manner as shown in the third and fourth examples.

Application to cases where the load is to be uniformly distributed over the length of the beam.

18. Whether a load be uniformly distributed over the length from A to B, *Fig.* 2, *Plate* I. or it be collected at several equidistant points, as at 1, 2, 3, 4, 5, 6, and 7, in the same figure, the same rule may be used, as it causes no difference that need be regarded in practice.

It is proved by writers on the resistance of solids, that the whole of the load upon a beam when it is uniformly distributed over it, will produce the same degree of deflexion as five-eighths of the load applied in the middle*, (see experiment, art. 43, 50, and 51.) Consequently take five-eighths of the whole load upon the beam, and with this reduced weight proceed as in the foregoing examples.

* Dr. Young's Lectures on Nat. Phil. vol. ii. art. 325, 329. Mr. Barlow's Essay on the Strength of Timber, art. 91.

19. Example IX. Let it be required to find the dimensions of a cast iron bar to support ten tons uniformly distributed over its length, the depth of the bar to be four times its breadth, and the deflexion to be not more than one-eightieth part of an inch for each foot in length, or one-fourth of an inch, the length being twenty feet.

Here the five-eighths of ten tons is six tons and a quarter, and as the depth is to be four times the breadth, multiplying six and a quarter by four, gives twenty five tons; but the deflexion is to be only half that given in the table, therefore the twenty-five must be doubled, which gives fifty for the number of tons opposite which the depth is to be found. The depth opposite fifty tons, and under twenty feet, is 14·6 inches, and the breadth is $\frac{14 \cdot 6}{4}$ or 3·65 inches; that is, a bar 14·6 inches deep, and 3·65 inches in breadth, will bear a load of ten tons uniformly distributed over it when the length of bearing is twenty feet, and the deflexion in the middle a quarter of an inch.

When there is not any length and weight in the table exactly the same as those which are given, take the nearest; the dimensions thus obtained will always be sufficiently near for practice.

SECTION III.

OF THE FORMS OF GREATEST STRENGTH FOR BEAMS.

20. In the introduction, I have stated that one of the most valuable properties of cast iron, consists in our being able to mould it into the strongest form for our intended purpose; and in order to apply this property with the most advantage, it will be useful to consider the means of applying our theoretical knowledge on this subject to practice.

There are two means of increasing the strength of a beam, the one consists in disposing the parts of the cross section in the most advantageous form; the other, in diminishing the beam towards the parts that are least strained, so that the strain may be equal in every part of the length.

Of Forms of equal Strength.

21. Before I point out the forms of equal strength corresponding to different modes of applying the load or straining force, let us consider the conditions that are essential in a practical point of view. In the first place, supported parts must have sufficient mag-

nitude to ensure stability; for it is much more important that every connection or joining should be firm, and that the bearing parts should be secure against crushing or indentation, than it is that a small portion of material should be saved. When mathematicians investigate a form of equal strength, the manner of connecting it or supporting it, is not considered, therefore the forms given by them do not answer in practice.

22. If a beam be supported at the ends, and the load applied at some one point between the supports, and always acting in the same direction, the best plan appears to be to keep the extended side perfectly straight, and to make the breadth the same throughout the length; then the mathematical form of the compressed side is that formed by drawing two semi-parabolas ACD and BCD, *Fig.* 3, C being the point where the force acts*. Now the curve terminating at A, it is necessary in applying it to use, to add some such parts as are indicated by the dotted lines at the extremities.

23. Irregular additions of this kind, however, render it difficult to estimate the effect of the straining force; therefore, some simple straight-lined figure to include the parabolic form is to be preferred: this may be easily effected by making the lines bounding the compressed side tangents to the parabolas, as in *Fig.* 4. If AE be equal to half CD, then EC is a tangent to the parabola AC.

* Greg. Mechanics, 1. art. 179.

24. If the beam be strained sometimes from one side and sometimes from the other, both sides should be of the same figure, as in *Fig. 5*. In the beam of a double acting steam engine, the strain is of this kind. AE and BF should be equal, and each equal to half CD as before.

25. It is sometimes desirable to preserve the same depth throughout; and in this case, the section through the beam made perpendicular to the direction of the straining force should be a trapezium, described in the manner shown in the 6th figure *, the force acting perpendicularly at C, A, and B, being the points of support. A figure of this kind would obviously be without stability, but modified as shown by *Fig.* 7, the end being formed as shown at B′, any degree of stability may be given, and with a less quantity of material than when the depth is diminished, as in the parabolic form. Also, the deflexion is less, which gives this form a considerable advantage for bearing purposes.

26. If a weight be uniformly distributed over the length of a beam supported at both ends, and the breadth be the same throughout, the line bounding the compressed side should be a semi-ellipse when the lower side is straight †, as shown in *Fig.* 8.

Instead of an ellipse, I usually make the compressed side a portion of a circle, of which the radius is equal to the square of half the length divided by the depth of the beam. The dotted line in *Fig.* 8, shows this form.

* Greg. Mechan. I. art. 179. † Idem, I. art. 182.

SECTION IV.

OF THE STRONGEST FORM OF SECTION.

27. WHEN a rectangular beam is supported at the ends, and loaded in the middle between the supports, it may be observed that the side against which the force acts is always compressed, and that the opposite side is always extended; and that the middle of the depth is neither extended nor compressed; or, in other words, it is not strained at all. The strains decrease from each side towards the middle, and in the middle they are insensible.

I will call the part at the middle of the depth the neutral axis, or *axis of motion*. See Sect VI. art. 76.

28. In the case of equilibrium, between the straining force and the resistance of a beam, it is a necessary condition that the resistance on one side of the axis of motion should be exactly equal to the resistance on the other side; or, that the force of compression should be equal to the force of extension. And as it is known from experience, that while their elastic force remains perfect, bodies resist the same degree of extension or of compression with equal forces; it is obvious that in the section of a beam of the greatest strength, the form on each side of the

axis of motion should be the same. Hence, the axis of motion in beams of the greatest strength will always be at the middle of the depth.

29. And, as it is shown by writers on the resistance of solids, that the power of any part in the same section is directly as the square of its distance from the axis of motion, (art. 77.) when the strain upon it is the same, it is obviously an advantage to dispose the parts of the section at the greatest possible distance from the axis of motion, provided that the middle parts be kept sufficiently strong to prevent the straining force from crushing the extreme parts together, and that the breadth be sufficient to give stability.

30. It must also be observed, that when the parts are not of equal thickness, the metal cools unequally, and therefore is partially strained by irregular contraction; it is sometimes even fractured by such irregular cooling; for this reason, the parts of a beam should be nearly of the same size. A good founder may generally reduce the danger of irregular cooling, but it is always best to avoid it altogether.

31. The form of section which I usually adopt in order to fulfil these conditions, is represented in *Fig.* 9. AM is the axis of motion; the parts on each side of the axis of motion are the same; the metal is nearly of equal thickness, and the parts necessary to give breadth and stability, are disposed at the greatest distance from the axis of motion.

A section of this form is adapted for many purposes, such for example as the beam of a steam engine, or for supporting arches as in *Fig.* 10, and the like.

32. When it is necessary to leave some part of the middle of the beam quite open, or when the depth is considerable, I have recourse to another method, which has, in such cases, a decided advantage in point of economy. It consists in making the compressed side of the beam, or that against which the force acts, a series of arches, and the other side a straight tie. See *Fig.* 11, *Plate* II.

In this figure, the thickness is supposed to be every where the same, and the narrowest part of the curved side of the same width as the straight side; or, so that the area of the section at AB may be the same as the area of the section at CD.

The sketch in the figure is for the case in which the load is uniformly distributed over the length, and then the upper side should be the proper curve of equilibrium for an uniform load. This curve is a common parabola, but a circular arc will always be sufficiently near when the rise is so small. It forms an arch, of which the continued tie forms the abutments, and the smaller arches are merely to connect the two parts and give stability to the whole.

All the parts should be kept as nearly as convenient of the same bulk, to prevent irregular contraction.

33. If the load be distributed in any other manner, the curve should be the proper curve of equilibrium for that load*.

* The method of finding the curve of equilibrium is shown in my "Elementary Principles of Carpentry," Sect. I. art. 47 to 61.

34. If the load be applied at one point, the upper side should be formed of two straight lines, meeting in the point where the load is to rest, as at A in *Fig.* 12.

The openings should be disposed as may best answer the purpose for which the beam is intended, but they may generally be from two to three feet each. When such beams, as *Fig.* 11. are used as girders, the openings receive the binding joists instead of mortices.

Of the strongest Form of Section for revolving Shafts.

35. When a beam revolves, while the straining force continues to act in the same direction upon it, that form is obviously the best which is of the same strength to resist a stress at any point of the perimeter of its section, and the circle is the only form of section which has this property.

If a shaft be of any other form than cylindrical, the flexure will be different in different parts of the revolution, and therefore the motion will be unsteady, and particularly in new work. In a square shaft, the flexure from a pressure at one point is to the flexure from the same pressure at another point in the perimeter as ten is to seven nearly. In feathered shafts, that is, shafts of which the section is similar to *Fig.* 13, the flexure is more regular, but not perfectly so*.

* In heavy Astronomical Instruments, and in all machines where steady and accurate movements are necessary, every attention should

36. As the circle is the best form for the section of a shaft, a hollow cylinder will be the strongest and stiffest form for a shaft; and the same form is also best calculated for resisting a twisting strain to which all shafts are more or less exposed.

The idea of making hollow tubes for resisting forces that often change their direction, has been undoubtedly borrowed from nature; but in art we cannot pursue the principle to so much advantage, because it is difficult to make a perfect casting of a thin tube, and in shafts, &c. of small diameter, it is much greater economy to make them solid.

It is usual to make hollow tubes of uniform diameter with gudgeons cast separate, to fix at the ends. The manner of calculating the stiffness of hollow tubes, for shafts, is shown in art. 219 and 220. When they are applied to other purposes consult art. 140, and those following it in the same proposition.

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be paid to the effect of flexure. Irregularity may be diminished by excess of strength, but it cannot be wholly removed.

SECTION V.

AN ACCOUNT OF SOME EXPERIMENTS ON THE RESISTANCE OF CAST IRON.

37. THERE have been very few experiments made on the resistance of cast iron, in which the degree of flexure produced by a given weight has been measured; but the few that have come to my knowledge, and that are sufficiently described to admit of comparison, I purpose to compare with the rules I made use of in calculating the tables in this work; and to add several new experiments.

*Mr. Banks's Experiments.**

38. Mr. Banks made some experiments on cast iron, and noticed the deflexion, but only at the time of fracture. These experiments were made at a foundry at Wakefield. The iron was cast from the air-furnace; the bars one inch square, and the props

* From a treatise " On the Power of Machines," by John Banks. Kendall, 1803, p. 96.

exactly a yard distant. One yard in length weighed exactly 9lbs. excepting one, which was about half an ounce less, and another a very little more. They all bent about an inch before they broke.

1st bar broke with	963 lbs.	Mean 971⅔ lbs.
2d bar broke with	958 —	
3d bar broke with	994 —	
4th bar, made from the cupola, broke with	864 —	

39. Now the rule according to which the first table was calculated is expressed by the equation $\cdot 001 WL^2 = BD^3$, in which the weight in pounds is denoted by W, the length in feet by L, the breadth in inches by B, the depth in inches by D, and the number ·001 is a constant multiplier, which I shall sometimes denote by *a*.

The rule determines the dimensions for a deflexion of as many fortieths of an inch as there are feet in length, or $\frac{L}{40}$; and if *d* be the deflexion in inches determined by experiment, we have $d : W :: \frac{L}{40} : \frac{WL}{40d}$, which being substituted for the weight in the equation above it, becomes

$$\frac{\cdot 001 WL^3}{40d} = BD^3.$$

$$\text{Or, } \cdot 001 = a = \frac{40BD^3d.}{WL^3}$$

The equation, in this form, may be called a formula of comparison, as when the value of *a* determined by it is the same I have used, or nearly the

same, it will be evident that the table is calculated from proper data.

40. Taking the mean of the first three of Mr. Banks's experiments, we have

$$\frac{40BD^3d}{WL^3} = \frac{40}{971 \times 27} = \cdot 00152 = a.$$

And in the bar from the cupola, or fourth experiment,

$$\frac{40BD^3d}{WL^3} = \frac{40}{864 \times 27} = \cdot 0017 = a.$$

The experiments of Mr. Banks are not sufficiently accurate for establishing the elements of a practical rule, because the deflexion was not correctly observed, nor observed at a proper stage of the experiment. For when a bar is strained nearly to the point of fracture, the deflexion becomes extremely irregular, and increases more rapidly than in the simple proportion of the weight; (see art. 45, 52, 54, and 56) consequently must give a much higher value to a than the true one, as we find to be the case with these experiments.

*Mr. Rondelet's Experiments.**

41. Mr. Rondelet has described some experiments on different kinds of cast iron in his work on building, which were made upon specimens of 1·066 inches square, supported at the ends, and loaded in the middle of the length.

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* Extracted from his Traité Théorique et Practique de L'Art de Bâtir, 6 tomes, 4to, Paris, 1814, tome iv. p. 514.

Mr. Rondelet's First Experiments. Distanoe between the Supports 3·83 *feet.*

Weight in lbs.	134	201	268	335	Remarks, &c.
Kind of Iron.	Deflex. inches.	Defl. inch.	Defl. inch	Deflexion in inches.	
1. Gray cast iron	·089	·2	·357	·49	Broke with 482 lbs.
2. Do. Do.	·156	·313	·38	·49	Broke with 482 lbs.
				2) ·98(	·49 mean of deflexion, with 335 lbs.
3. Soft cast iron	·134	.313	·466	·62	Broke with 700 lbs.
4. Do. Do.	.0223	·067	·134	·2	Broke with 1140 lbs.
5. Do. Do.	·089	.156	·245	·38	Broke with 375 lbs.
6. Do. Do.	·089	·178	·29	·445	Broke with 605 lbs.
				4)1·645(	·411 mean of deflexions, with 335lbs.

Mr. Rondelet's Second Experiments. Distance between the Supports 1·915 *feet.*

Weight in lbs.	322	483	644	805	Remarks, &c.
Kind of Iron.	Deflex. in inch.	Deflex. in inch.	Deflex. in inch.	Deflex in inch.	
1. Gray cast iron	·067	·089			Broke with 580 lbs.
2. Do. do.	·0445	·089	·112	·134	Broke with 1063 lbs.
					Mean of deflexions, with 483 lbs. is ·089 inches.
3. Soft cast iron	·0445	·089	·134	·153	Broke with 1770 lbs.
4. Do. do.	·0445	·067	·134		Broke with 1360 lbs.
		2)·156			Mean of deflexions, with 483 lbs. is ·078 inches.
		·078			

In order to compare these results with the formula used in calculating the table, I have taken the

mean deflexions corresponding to the load of 335 lbs. in the long pieces, and to 483 lbs. in the short ones; and in the gray cast iron,

For the long lengths......... a = ·00134

For the short lengths ...... a = .00135

In the soft cast iron,

For the long lengths ...... a = ·00112

For the short lengths ...... a = ·00118

These values of a were calculated by the formula of comparison given in art. 39.

Mr. Ebbels's Experiment.

42. According to a trial communicated to me by Mr. R. Ebbels, a bar of cast iron, one inch square, and supported at the ends, the distance of the supports being three feet, the deflexion in the middle was three-sixteenths of an inch, with a weight of 308 lbs. suspended from the middle. The iron was of a hard kind, not yielding very easily to the file; it was cast at a Welsh foundry.

In this trial we have

$$\frac{40BD^3d}{L^3W} = \frac{40\times 3}{27\times 308\times 16} = \cdot 000902 = a.$$

Consequently, iron of this kind is about one-tenth stronger than that which the table is calculated from, or rather it would bend one-tenth part less under the same strain.

Experiment 1.

43. A joist of cast iron of the form described in *Fig.* 9. *Plate* I. was submitted to the following trials. It was supported at the ends only; the distance

between the supports nineteen feet, and placed on its edge. The deflexion from its own weight was three-fortieths of an inch.

When it was laid flat-wise, the deflexion from its own weight was 3·5 inches, the distance of the supports remaining nineteen feet.

The whole depth *ad*, *Fig.* 9. was nine inches, the breadth, *bb*, was two inches; the depth of the middle part, *ef*, was seven inches and a half; and the breadth of the middle part three quarters of an inch.

44. It may be easily shown that to derive the value of a, from the experiment on the edge, we may use an equation of this form, (see art. 154 and 177.)

$$a = \frac{40BD^3d(1-p^3q)}{4WL^3} = \frac{64BD^3d(1-p^3q)}{WL^3};$$

in which D is the whole depth, and pD the depth of the middle part, and B the whole breadth, and qB the breadth after deducting that of the middle part.

In our experiment D $=$ 9 inches, and pD $=$ 7·5, or $p = \cdot 83\dot{3}$. Also, B $=$ 2 inches, and deducting three-fourths, the breadth of the middle, we have qB $=$ 1·25, or $q = \cdot 625$. And the weight of the part of the joist between the supports being 540 lbs. we find $a = \cdot 00124\cdot$

The equation for finding the value of a in the experiment with the joist, flat-wise, is

$$\frac{64BD^3d(1+p^3q)}{WL^3} = a = \cdot 00092. \text{ Where } D = 2 \text{ inches,}$$

$$B = 9 - 7\cdot 5 = 1\cdot 5, p = \frac{\cdot 75}{2}, \text{ and } q = \frac{7\cdot 5}{1\cdot 5}.$$

I consider the value of a derived from the expe-

riment with the joist flat-wise as nearest the truth, because the deflexion was so considerable, that a small error in measuring it would not sensibly affect the result, while there must be some uncertainty in ascertaining so small a deflexion as three-fortieths of an inch in nineteen feet; and a very small error in this measure would cause the difference between the results. I have, however, given it as I determined it at the time, and the manner of calculation may be useful in other cases. If the mean be taken between the results, it is $\frac{\cdot00124 \times .00092}{2} = \cdot00108$. In the experiment, flat-wise, we obtain a constant multiplier extremely near to that determined from a bar of the same iron an inch square and thirty-four inches long, (art. 46.) and it differs only about one-twelfth part from the one employed for calculating the table, page 8, art. 5.

Experiment 2.

45. I now purpose describing the direct experiments I have made for obtaining the constant multipliers used in this work; I call experiments direct when known weights are applied, as the straining force without the intervention of mechanical powers, without loss of effect from friction, or a risk of error in estimating the quantity of force, when the yielding of the supports cannot affect the measure of the deflexion, and when the deflexion can be accurately measured.

The iron I used was soft gray cast iron, it yielded

easily to the file, and extended a little under the hammer, before it became brittle and short.*

The first experiment was made with a bar of an inch square, cast by Messrs. Dowson, London, with the supports thirty-four inches apart; the weights were placed in a scale suspended from the middle of the length; the load was increased by 10 lbs. at a time, and the deflexion measured each time, the quantity of deflexion being multiplied by means of a lever index. The whole time of making the experiment was nearly four hours; the thermometer varying from sixty-five to sixty-six degrees. Only half the number of observations is inserted here.

Weight in lbs.	Defl. in In.	Remarks.	Weight in lbs.	Defl. in In.	Remarks.
20†	·02		240	·13	
40	·03		260	·14	
60	·04		280	·15	unloaded, and it returned to its natural state.
80	·05		300	·16	
100	·06		320	·17	
120	.07		340	·18	
140	·08		360	·19	
160	·09		380	·2	
180	·10	unloaded, and it returned to its natural state.	400	·21	
200	·11		410	·22	deflexion became irregular; and when the load was removed, it had taken a permanent set, with a curvature of ·015 inches.
220	·12				

From this experiment we find that the deflexion

* A considerable degree of malleability is a good quality in cast iron for bearing purposes, because it lessens the risk of sudden failure.

† The weight of the scale, 8lbs. ought to have been added.

of cast iron is exactly proportioned to the load, till the strain arrives at a certain magnitude, and it then becomes irregular; and at or near the same strain a permanent alteration takes place in the structure of the iron, and a part of its elastic force is lost. The same thing occurs in experiments on other metals: I have tried tin, zinc, lead, and alloys of tin and lead, with a view to measure their elastic forces, and the strains that produce permanent alteration.

46. According to this experiment, $\frac{40BD^3d}{WL^3} = \frac{40 \times \cdot 21}{400 \times 22\cdot 7} = \cdot 000925 = a.$

Experiment 3.

47. The next experiments were made with a uniform bar of iron, cast by Messrs. Dowson, three inches by one and one half inches, and 6.5 feet between the supports. When this bar was placed on its edge, and loaded in the middle with

150 lbs. the deflexion in the middle was	1	fortieth of an inch.
290 lbs. ———	2	do.
360 lbs. ———	$2\frac{1}{7}$	do.
440 lbs. ———	3	do.

The same deflexions were observed in removing the load, and it perfectly regained its natural state. Whence we have,

$$\frac{40BD^3d}{WL^3} = \frac{1\cdot 5 \times 27 \times 3}{440 \times 274\cdot 625} = \cdot 00105, \text{ nearly} = a.$$

Experiment 4.

48. The same piece, with the supports at the

same distance, placed flat-wise, and loaded in the middle, with

180 lbs. the deflexion in the middle was 5 fortieths of an inch.
360 lbs. ———————————— 10 do.

The bar restored itself perfectly when the weights were removed, and the trial was repeated with the same results; the load, of 360lbs. remained upon it ten hours without impairing its elastic force, or increasing the deflexion in the slightest degree.

49. From this and the preceding experiment, the ratio of the breadth and depth to the quantity of deflexions may be compared when the weight is the same. According to the theory of the resistance to flexure, (art. 216,) $d : \frac{1}{BD^3}$; and to the weight of 360 lbs. we have

$\frac{1}{1 \cdot 5 \times 3^3} : \frac{1}{3 \times 1 \cdot 5^3} :: 2\frac{1}{2} : \frac{9 \times 2 \cdot 5}{2 \cdot 25} = 10$, as it was found to be by experiment.

To find the constant multiplier from the last experiment, we have

$$\frac{40BD^3d}{WL^3} = \frac{3 \times 3 \cdot 375 \times 10}{360 \times 274 \cdot 625} = \cdot 00102 = a.$$

This value of *a* does not exactly agree with the one calculated from the first experiment on the same piece; but it is as near as can be expected in a case of this kind; and in a practical point of view it is as near an approach to accuracy as the nature of the subject requires.

Experiment 5.

50. I was desirous of trying the effect of an uniformly distributed load, and my weights, which are

cubical pieces of cast iron, all of the same size, and each weighing ten pounds, are very well adapted for the purpose.

The same piece that was used for the last experiment was laid flat-wise upon supports, the supports being 6 feet 6 inches apart, and 18 weights (in all 180 lbs.) were laid along the upper side, just so as to be clear of one another, in the manner shown in *Fig.* 2. *Plate* I. The deflexion produced by these weights was 3-40ths of an inch.

A second tier of weights being added, making the whole weight upon the bar 360lbs. the deflexion was 6-40ths of an inch.

51. Hence it appears, that when the weight is uniformly distributed over the length, the deflexion is directly as the weight.

And comparing this with the preceding experiment, it appears, that the deflexion from the weight uniformly distributed over the length, is to the deflexion from the same weight applied in the middle of the length, as 6 is to 10.

The proportion obtained by theoretical investigation is as 5 is to 8; but as 6 : 10 :: 5 : 8⅓. This small difference arises undoubtedly from error in measuring the deflexions in the experiments.

To compare the value of the constant multiplier by this experiment, the equation $\frac{40BD^3d}{\frac{5}{8}WL^3} = a$ must be used, whence we find $a = \cdot 00098$.

Experiment 6.

52. This experiment was made upon a piece of iron, cast by Messrs. Bramah, of Pimlico, London. It crumbled sooner under the hammer than that used in the preceding experiments, and did not yield quite so readily to the file; it was regular and fine-grained.

The piece was uniform, and 9-10ths of an inch square; the supports were three feet apart, and the weight was applied in the middle of the distance between the supports.

Weight in lbs.	Defl. in In.	Remarks.	Weight in lbs.	Defl. in In.	Remarks.
20	·02		220	·225	
40	·04		240	·245	
60	·06	When unloaded it returned to its original form; loaded again the deflexion was the same, and it remained loaded 12 hours without sensible increase, when on being unloaded it was found to have acquired a permanent set of ·02 in. The index was set to nothing, and the weights produced the same deflexions as at first; and it was further loaded as described.	260	·27	
80	·08		280	·293	
100	·0		300	·318	When this load had been on 20 minutes, it became ·32 inches.
120	·12		320	·34	
140	·14		340	·365	
160	·162		360	·392	
180	·183		380	·42	
200	·21		400	·445	
			420	·475	
			440	·5	
			460	·532	
			480	·57	which became in an hour ·58.

When the weights were removed, the piece retained a permanent deflexion of ·075 inches; but it was several hours before it returned to that curvature. I did not break the specimen, because I had not weight enough by me for that purpose, neither

would it have given a fair measure of the strength of the iron after the trials I have described; but I hope the effect of these trials will make the reader sensible of the necessity of limiting the strain within the range of the elastic force of the material.

According to this experiment

$$\frac{40BD^3d}{WL^3} = \frac{40 \times \cdot 9^4 \times \cdot 21}{200 \times 27} = \cdot 00102 = a.$$

Comparison of the preceding Experiments.

53. If the mean value of the constant a be taken for the Experiments from art. 42 to 52, it is 0·0010446. The number used in calculating the first table (art. 5. p. 8.) was 0·001, a sufficiently near approximation, with the advantage of much simplicity.

Experiments 7, 8, *and* 9.

54. The next trials were made with specimens formed as shown in *Fig.* 4. *Plate* I. with the deepest part CD, exactly in the middle of the length, and the depth at CD 0·975 inches, the depth EA and BF were each half that at CD. The distance of the supported points AB, was three feet, and the breadth of the bars 0·75 inches. The load was suspended from the point C in the middle of the length, and the deflexion was measured at the same point: the load was increased by 10 lbs. at a time.

Weight acting on the bar.	1st Specimen. Deflex. produced.	2nd Specimen Deflex. produced.	3rd Specimen Deflex. produced.
40 lbs.	·052 inches.	·065 inches.	·052 inches.
80 —	·104 —	·13 —	·105 —
120 —	·16 —	·19 —	·16 —
160 —	·215 —	·25 —	·21 —
180 —	·245 —	·28 —	·24 —
200 —	·272 —	·32 —	·265 —
500 —	·84 —		
640 —	Broke.		

On the first specimen the load of 180lbs. remained 12 hours; the deflexion did not sensibly increase, and it returned to its natural form when unloaded: it was again loaded to 200 lbs. which remained upon it two hours; it was then unloaded again, and was found to have taken a permanent set with a deflexion of ·005 inches. The specimen was then loaded again, and the deflexions observed at every 20lbs.: the deflexion produced by the addition of 20lbs. was at first ·026 became ·03, ·04, and towards the end of the experiment ·05. When the load had been increased to 360lbs. in every succeeding addition of 10 lbs. I observed that the deflexion increased by starts of as much as $\frac{1}{100}$ of an inch each, which appeared to be caused by the ends sliding on the supports, at the moment the weight was added; the bar emitted a slight crackling noise like that produced by bending a piece of tin. There was a small defect in the bar at the place where it broke, which was four inches distant from the middle.

When the second specimen was unloaded im-

mediately, from a weight of 200lbs. it barely returned to its natural form; but a load of 180lbs. produced a permanent deflexion of ·005 when it remained upon it 14 hours.

The load of 200lbs. remained 21 hours upon the third specimen, and when it was unloaded the index returned to zero; therefore this strain was less than would produce a permanent set. The set was nearly ·01 when the load was increased to 210lbs. and remained upon it 10 hours. It was a smoother and better casting than the other specimens.

There did not appear to be any sensible difference in the quality of the iron in these specimens, except that the second specimen was more brittle under the hammer than the other two. They were all fine grained, and yielded easily to the file. They were cast by Messrs. Bramah.

55. I was proceeding with a trial of a piece of the same kind of iron, formed as described in *Fig.* 4. *Plate* I. when it broke suddenly, at about a foot from the end, at an air bubble. The bubble was not apparent on the surface, aud yet so near it, that a slight stroke of a hammer would have broken into it. Founders should be very careful to avoid defects of this kind; and beams to sustain great weights should always be proved to a deflexion within their range of elasticity before they be used.

Experiments 10, 11, *and* 12.

56. These trials were made on three pieces of uniform breadth and depth, with the supports three

feet apart, the load being applied in the middle of the length. The depth .9 inches, and the breadth the same.

Weight acting on the bar.	1st Specimen. Deflex. produced.	2nd Specimen. Deflex. produced.	3rd Specimen. Deflex. produced.
40 lbs.	·041 in.	·042 in.	.041 in.
80 —	·082 —	.09 —	·08 —
120 —	·124 —	·136 —	·12 —
160 —	·165 —	·18 —	·16 —
180 —	·185 —	·202 —	·18 —
200 —	·206 —		20 —

The load of 200 lbs remained 12 hours on the first specimen, and when it was unloaded the quantity of permanent deflexion was barely sensible; and it was loaded and unloaded again with the same result.

The load of 180lbs. remained three hours on the second specimen; it had not increased the deflexion, but when the load was removed, it was found that the bar had acquired a permanent set of nearly 1-100th of an inch.

In the third specimen the bar returned perfectly to its natural form when the load was removed after being upon it three hours.

Of these specimens the third was the most brittle under the hammer, and the hardest to the file; there was not a sensible difference between the other two; both were soft iron. These specimens were cast by Messrs. Bramah.

57. The chief object in view in the Experiments No. 2, 6, 7, 8, 9, 10, and 11, was to determine the

strain a square inch of cast iron would bear without permanent alteration, and the extension corresponding to that strain. Calling f this strain in lbs. the Experiment 2, gives $f = 15,300$ lbs. as calculated in art. 105, and the others being calculated by the same formula, in Experiment 6, 10, and 12, $f = 14,814$; in Experiments 7, 8, and 9. $f = 15,160$; and in Experiment 11, $f = 13,333$ lbs. The greatest difference amounts to about one-eighth of the highest value of f; but, in the Experiment 2, the load was taken off after remaining only about ten minutes on the bar; in the others it remained on for several hours. The former I consider most strictly applicable to practice; and yet it was desirable to show, that a force acting a considerable time, will produce a permanent set, when the same force could not produce it in a few minutes.

58. In art. 156, it is calculated that the extension produced by the strain of 15,300 lbs. in Experiment 2, was $\frac{1}{1204}$ of the length; and by the same mode of calculation the extension in Experiment 6, is found to be $\frac{1}{1143}$, in Experiment 10, $\frac{1}{1165}$, in Experiment 11, $\frac{1}{1170}$, and in Experiment 12, $\frac{1}{1200}$. Also, by the equation, art. 92. the extension in Experiment 7, is found to be $\frac{1}{1332}$, in Experiment 8, $\frac{1}{1132}$, and in Experiment 9, $\frac{1}{1367}$.

The difference between the extension in the 8th

and 9th Experiments is the most considerable; and the mean between these is $\frac{1}{1239}$, which differs very little from $\frac{1}{1204}$, the number used in the rules.

59. A table of the chief experiments that have been made on the absolute strength of cast-iron bars to resist a cross strain, the bars supported at the end, and loaded in the middle.

No.	Description.	Length between the supports in feet.		Dimensions at the strained point in inches.		Weight in lbs. that broke it.	Calculated wt. that would destroy elastic force in lbs.	Ratio of the calculated wt. to the breaking weights.
		ft.	in.	brdth.	dpth.			
1	Uniform bar	3	0	1	1	756	283	1:2·7
2	Ditto	3	0	1	1	735	283	1:2·6
3	Ditto	2	6	1	1	1003	340	1:2·96
4	Ditto	3	0	1	1	963	283	1:3·4
5	Ditto	3	0	1	1	958	283	1:3·38
6	Ditto	3	0	1	1	994	283	1:3·5
7	Ditto cast from the cupola	3	0	1	1	864	283	1:3·05
8	Parabolic bar cast from the cupola	3	0	1	1	874	283	1:3·08
9	Uniform bar	3	0	1	1	897	283	1:3 17
10	Ditto	2	8	1	1	1086	318·75	1:3·4
11	Ditto	1	4	1	1	2320	637·5	1:3·6
12	Ditto	2	8	2	½	2185	637·5	1:3·42
13	Ditto	1	4	2	½	4508	1275	1:3·53
14	Ditto	2	8	3	⅓	3588	956·25	1:3·63
15	Ditto	1	4	3	⅓	6854	1912·5	1:3·58
16	Ditto	2	8	4	¼	3979	1275	1:3·12
17	Semi-ellipse	2	8	4	¼	4000	1275	1:3·14
18	Parabolic	2	8	4	¼	3860	1275	1:3·03
19	Uniform strain in the direction of diagonal	2	8	$\sqrt{2}$	$\sqrt{2}$	851	224·5	1:3·79

In the preceding table, the Experiments 1, 2, and 3, were made by Mr. Reynolds. No. 1. was twice repeated with the same result. No. 2. is a mean of three experiments*. The Experiments, No. 4, 5, 6, 7 and 8, were made by Mr. Banks †. The rest were made by Mr. George Rennie, and all of the bars of his experiments were cast from the cupola ‡.

The two columns on the right hand side are added to show the relation between the load which permanently destroys a part of the elastic force, and that which breaks the piece. It will be seen that the load which would produce permanent alteration, according to the formula as derived from my experiments, is about one-third of that which actually broke the specimens; in the worst kind tried, it is $\frac{1}{2{\cdot}6}$ of the breaking weight.

Experiments on the Resistance to Tension.

60. According to an experiment made by Muschenbroëk, a parallelopipedon, of which the side was ·17 of a Rhinland inch, broke with 1930 lbs.§; and since the Rhinland foot is 1·03 English feet, and the pound contains 7038 grains, this experiment gives 63,286 lbs. for the weight that would tear asunder a

* Banks on the Power of Machines, p. 89.

† Idem p. 90.

‡ Philosophical Transactions, for 1818, Part I. or Philosophical Magazine, vol. liii. p. 173.

§ Muschenbroëk's Intro. ad Phil. Nat. vol. i. p. 417, 1762.

square inch, when reduced to English weights and measures.

61. An experiment made by Capt. S. Brown, is thus described; " A bar of cast iron, Welsh pig, $1\frac{1}{4}$ inch square, 3 feet 6 inches long, required a strain of 11 tons 7 cwt. (25,424 lbs.) to tear it asunder: broke exactly transverse, without being reduced in any part; quite cold when broken; particles fine, dark bluish grey colour *."

Capt. Brown's machine for trying such experiments being constructed on the principle of a weighbridge, Mr. Barlow is of opinion it may show less than its real force; it also may be remarked that to obtain the real force of cohesion, the resultant of the straining force should coincide exactly with the axis of the piece; for so small a deviation in this respect as one-sixth of the breadth would reduce the strength one half.

From this experiment it appears that 16,265 lbs. will tear asunder a square inch of cast iron.

62. In some experiments made by Mr. G. *Rennie*, it is obvious from the description of the apparatus, that the strain on the section of fracture would not be equal; and, therefore, that the straining force would be less than the cohesion of the section. The specimens were 6 inches long, and $\frac{1}{4}$ of an inch square at the section of fracture. A bar cast horizontally, required a force of 1166 lbs. to tear it asun-

* Essay on the Strength of Timber, &c. by Mr. Barlow, 1817, p. 235.

der. A bar cast vertically, required a force of 1218 lbs. to tear it asunder*.

In the horizontal casting the force was equal to	18,656	pounds per square inch.
and in the vertical casting......	19,488	———

Experiments on the Resistance to Compression in short Lengths.

63. The power of cast iron to resist compression, was formerly much over-rated. Mr. Wilson estimated the power necessary to crush a cubic inch of cast iron at 1000 tons=2,240,000 lbs. Mr. Reynolds of Colebrook-Dale, estimated that a cube of $\frac{1}{4}$ of an inch of cast iron, of the quality called gun-metal, requires 448,000 lbs. to crush it †; and consequently a cubic inch would require 7,168,000 lbs.=3,200 tons to crush it; above three times Mr. Wilson's estimate; and thirty-five times the highest result of later experiments.

64. Such was the state of our knowledge, on this important subject, when Mr. G. Rennie communicated a valuable series of experiments to the Royal Society, which were published in the first part of their Transactions for 1818.

* Phil. Transactions for 1818, Part I. or Philosophical Magazine vol. liii. p. 167.

† Edin. Encyclo. art. Bridge; or Nicholson's Journal, vol. xxxv. p. 4, 1813.

Mr. Rennie's experiments on cubes from the middle of a large block, specific gravity 7.033.

	Force per sq. in. in lbs.
Side of cube $\frac{1}{8}$ in. was crushed by 1,454lbs. highest result	= 93,056.
Ditto —— $\frac{1}{8}$ do. ———— 1,416 — lowest do.	= 74,624.
Ditto —— $\frac{1}{4}$ do. ———— 10,561 — highest do.	= 168,976.
Ditto —— $\frac{1}{4}$ do. ———— 9,020 — lowest do.	= 144,320.

On cubes from horizontal castings, specific gravity 7·113.

	lbs. per sq. in.
Side of cube $\frac{1}{4}$ in. was crushed by 10,720lbs. highest result	= 171,520.
Ditto —— $\frac{1}{4}$ do. ———— 8,699 — lowest do.	= 139,184.

On cubes from vertical castings. specific gravity 7,074*.

Side of cube $\frac{1}{4}$ in. was crushed by 12,665lbs. highest result	= 202,640.
Ditto —— $\frac{1}{4}$ do. ———— 9,844 — lowest do.	= 157,540.

On pieces of different lengths

	lbs. per sq. in.
Area $\frac{1}{8} \times \frac{1}{8}$ length $\frac{1}{4}$ in. was crushed by 1,743lbs.	= 111,552.
Ditto $\frac{1}{8} \times \frac{1}{8}$ —— 1 do. ———— 1,439.	= 92,096
Ditto $\frac{1}{4} \times \frac{1}{4}$ —— $\frac{1}{2}$ do. ———— 9,374.	= 149,984.
Ditto $\frac{1}{4} \times \frac{1}{4}$ —— 1 do. ———— 6,321.	= 101,136.

These experiments were on too small a scale to allow of that precision in adjustment which theory shows to be essential in such experiments; therefore there still remains much to be done *by future* experimentalists. It does not appear, within the limits of these experiments, that an increase of length had any sensible effect on the result.

I have selected the highest and lowest results, and such of the single trials as were made under the greatest difference of length; in all Mr. Rennie made 39 trials on the resistance of cast iron to compression†.

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* It is singular that the specific gravity of the vertical castings should be less than that of the horizontal ones.

† Philosophical Transactions for 1818. Part I. Or Philosophical Magazine, vol. liii. p. 164, 165.

Experiments on the Resistance to Compression of Pieces of considerable Length.

65. The only experiments of this kind that I know of, were made by Mr. Reynolds, and are described as follows in Mr. Banks's work on the "Power of Machines," p. 89.

"Experiments on the strength of cast iron, tried at Ketley, in March, 1795. The different bars were all cast at one time out of the same air furnace, and the iron was very soft, so as to cut or file easily."

"Exp. 1. Two bars of iron, one inch square, and exactly three feet long, were placed upon an horizontal bar, so as to meet in a cap at the top, from which was suspended a scale; these bars made each an angle of 45° with the base plate, and of consequence formed an angle of 90° at the top; from this cap was suspended a weight of 7 tons, (15,680 lbs.) which was left for 16 hours, when the bars were a little bent, and but very little."

"Exp. 2. Two more bars of the same length and thickness, were placed in a similar manner making an angle of 22½° with the base plate; these bore 4 tons (8960 lbs.) upon the scale: a little more broke one of them which was observed to be a little crooked when first put up."

66. By the principles of statics 2 sin. 45° : Rad. : : 15,680 lbs. : 11,087 lbs.=the pressure in the direction of either bar, in the first experiment. And, 2 sin. 22½°: Rad. : : 8960 lbs. : 11,709 lbs. the pressure in the direction of either bar in the second experiment.

If we consider the direction of the force to have been exactly in the axis, in these trials, then according to the equation, art. 242, the greatest strain in the direction of one of these bars should not have exceeded 5840lbs.; but if the direction of the pressure was at the distance of half the depth from the axis, which it is very probable it would be, the greatest strain in actual construction should not have exceeded 2720lbs. See Sect. 6. art. 241.

Experiments on the Resistance to Twisting.

67. Table of the principal experiments of the strength of cast iron to resist a twisting strain.

No.	Description.	Leverage.	Length.	Side or Diameter in Inches.	Weight that broke the piece.	Calculated resistance without destroying the elastic force.	Ratio of the calculated resistance to the breaking weight.
1	Bar placed vertically, fast at one end and twisted by a wheel at the other.	2 feet	not given	1 × 1	631 lbs.		
		ft. in.	in.	in.			
2	Cylinder, fixed at one end, twisted by a lever at the other.	14 2	$2\frac{3}{4}$	2	250	73·7	1 : 3·39
3	Ditto	14 2	$3\frac{1}{4}$	$2\frac{1}{4}$	384	111	1 : 3·46
4	Ditto	14 2	3	$2\frac{1}{2}$	408	140	1 : 2·9
5	Ditto	14 2	3	$2\frac{3}{4}$	700	184	1 : 3·8
6	Ditto	14 2	4	$3\frac{1}{4}$	1170	309	1 : 3·78
7	Ditto	14 2	5	$3\frac{1}{2}$	1240	402	1 : 3·08
8	Ditto	14 2	5	$3\frac{3}{4}$	1662	481	1 : 3·45
9	Ditto	14 2	5	4	1938	580	1 : 3·34
10	Ditto	14 2	6	$4\frac{1}{2}$	2158	713	1 : 3·02

The experiment No. 1, was made by Mr. Banks*. The others were made by Mr. Dunlop, of Glasgow: No. 4, and 7, were faulty specimens†. Some experiments on a very small scale were made by Mr. George Rennie, not inserted here because they were not sufficiently described to admit of comparison‡.

Experiments on the Effect of Impulsive Force.

68. The height from which a weight might fall upon a piece of cast iron without destroying its elastic force was calculated by Equation 5. art. 264, for the specimens of ·9 inches square, used in the preceding experiments (art. 56). Repeated trials with that height of fall were made without producing a sensible effect. I then let the weight fall from double the calculated height, and every repetition of the blow added about 1-100th of an inch to the curvature of the bar. I could not measure the effect of each trial very correctly, but a few trials rendered the bar so much curved as to be easily seen. I hope, at some future time, to be able to resume these experiments with an apparatus for measuring correctly the degree of permanent set. See Sect. 7, where practical rules will be found.

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* "Power of Machines."

† Dr. Thomson's Annals of Philosophy, vol. xiii. p. 200—203.

‡ Philosophical Magazine, vol. liii. p. 168.

SECTION VI.

OF THE STRENGTH AND DEFLEXION OF CAST IRON WHEN IT RESISTS PRESSURE OR WEIGHT.

69. THE doctrine of the Strength of Materials, as given in this work, rests upon three first principles, and these are abundantly proved by experience.

The First is, That the strength of a bar or rod to resist a given strain, when drawn in the direction of its length, is directly proportional to the area of its cross section; while its elastic power remains perfect, and the direction of the force coincides with the axis.

70. The Second is, That the extension of a bar or rod by a force acting in the direction of its length, is directly proportional to the straining force, when the area of the section is the same; while the strain does not exceed the elastic power.

71. The Third is, That while the force is within the elastic power of the material, bodies resist extension and compression with equal forces.

72. It is farther supposed that every part of the same piece of the material is of the same quality, and that there are no defects in it. If there be any material defect in a piece of cast iron, it may often be discovered, either by inspection, or by the sound the piece emits when struck; except it be air-bubbles, which cannot be known by these means.

The manner of examining the quality of a piece of cast iron, has been given in the Introduction, p. 6; and such as will bear the test of hammering with the same apparent degree of malleability, will be found sufficiently near of the same strength and extensibility for any practical deductions to be correct.

The truth of these premises being admitted, every rule that is herein grounded upon them may be considered as firmly established as the properties of geometrical figures.

73. Let f denote the weight in pounds that would be borne by a rod of iron, of an inch square, when the strain is as great as it will bear without destroying a part of its elastic force*. Also, let W be any other weight to be supported, and b = the breadth and t = the thickness of the piece to support it, in inches. Then, by our first principles, art. 69. we have

$$f : W :: 1 : bt$$

$$\text{or, } \frac{W}{f} = bt \qquad \text{(i.)}$$

* "A permanent alteration of form," Dr. Young has remarked, "limits the strength of materials with regard to practical purposes,

71. If ϵ be the quantity a bar of iron, an inch square, and a foot in length, would be extended by the force f; and l be any other length in feet. Then

$$: l : : \epsilon : \Delta$$

or $l\epsilon = \Delta =$ the extension in the length l. (ii.)

For, when the force is the same, the extension is obviously proportional to the length.

And, since by our principle, art. 70.

f : W : : ϵ : to extension produced by the weight W, we obtain from Equation ii. $\frac{W l \epsilon}{f} = \Delta$. (iii.)

In which Δ is the extension that would be produced in the length l, by the weight W.

75. Let the rectangular beam A A′ *Fig.* 14. be supported upon a fulcrum D, in equilibrio, and for the present considering the beam to be acted upon by no other forces than the weights W,W′; which are supposed to have produced their full effect in deflecting the beam, and the vertical section at BD to be divided into equal, and very thin filaments, as shown in *Fig.* 15.

Consider B, *Fig.* 14. to be the situation of one of the small filaments in the upper part of the beam, and $a\,a'$ a tangent to the curvature of the filament B, at the point B. Now, it is clearly a necessary consequence of equilibrium that the forces tending to

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almost as much as fracture, since in general the force which is capable of producing this effect, is sufficient, with a small addition, to increase it till fracture takes place." Natural Philosophy, vol. i. page 141.

separate the filament at B should be equal, and in the direction of the tangent $a\,a'$; and the strain is obviously a tensile one.

But, since FA is the direction of the weight, we have, by the principles of statics, $B\,a : A\,a :: S$ (= the resistance of the filament B) : $\frac{A\,a.S}{B\,a}$ (= its effect in sustaining the weight W.)

These forces, we know both from reasoning and experience, will compress the lower part of the beam; and let D be a compressed filament, of the same area as the filament B, and in the same position, and at the same distance from the under surface as the filament B is from the upper surface. Also, let $e\,e'$ be a tangent to the filament at D, and parallel to $a\,a'$; and representing one of the equal and opposite strains on the filament D by $e\,D$; we have, $e\,D : e\,A :: S'$ (the resistance to compression of D) : $\frac{e\,A.S'}{e\,D}$ = the effect of the filament D in sustaining the weight W.

The effect of both the filaments, B, D, in supporting the weight will therefore be, $\frac{A\,a.S}{B\,a}+\frac{e\,A.S'}{e\,D}$, or since $B\,a = e\,D$, and as bodies of equal area resist extension or compression with equal forces (art. 71.) $S = S'$; therefore, $\frac{S}{B\,a}\times(A\,a + e\,A)$ = the effect of the filaments D and B. But $A\,a + e\,A = d$, the vertical distance between the filaments. Consequently $\frac{S.d}{B\,a}$ = this effect in supporting the weight W.

76. As the upper side of the beam suffers extension, and the lower side compression, there will be a filament at the middle of the depth, in a rectangular beam, which will neither be extended nor compressed; the situation of this filament may be called the neutral axis, or axis of motion.

77. When a beam is sustained in any position* by a fulcrum, as in Fig. 14. the power of a fibre or filament to support a weight at A or A′ is directly as its force, its area, and the square of its distance from the neutral axis; and inversely as the distance, FB, of the straining force from the point of support. For the strain being as the extension, and the extension of any filament being directly as the distance of that filament from the axis of motion, therefore, the force of a filament is as its distance from the axis of motion. But it has been shown, art. 73. that the force is also as the area; and the power in sustaining a weight has been shown, art. 75. to be directly as the vertical distance from the neutral axis, and inversely as the length, B a, that is, as $\frac{\mathrm{B}\,d}{\mathrm{B}\,a}$; and, since the triangles FB a, B df, are similar, $\frac{\mathrm{B}\,d}{\mathrm{B}\,a}=\frac{f\,d}{\mathrm{FB}}$ therefore $\frac{f d^2 \times \text{force of filament} \times \text{by its area}}{\mathrm{FB}}=$ the strain it will bear.

78. Let d be the vertical depth, divided into filaments, each equal to x the mth part of $\frac{d}{2}$; also put FB

* It does not sensibly differ from the correct law of resistance till the beam be so much inclined as to slide on its support.

$=l$, the breadth of the beam $= b$, and f the weight that a fibre of a given magnitude would bear when drawn in the direction of its length without destroying its elastic force.

Now, if we calculate the mean strain upon each filament by art. 77. we obtain the following progression, and its sum is the weight the beam will support.

$$\frac{2fbx^3}{ld} \times (\overline{0+1}+\overline{1+2^2} \pm \overline{2^2+3^2}+\overline{m-1^2+m^2}) = W.$$

79. If the beam be rectangular the value of $W = \frac{fbd^2}{6l}$. (iv.)

80. If the beam be square, and the strain be in the direction of its diagonal, making that diagonal $=a$, the progression becomes (because b is successively $\overline{a-2x}$, $\overline{a-4x}$, &c.)

$$\frac{2fx^3}{la} \times \left(a(\overline{0+1}+\overline{1+2^2}+\text{\&c.}) - 2x(\overline{0+1}+\overline{1+2^3}+\text{\&c.}\right) =$$

$$W = \frac{fa^3}{24l}. \qquad \text{(v.)}$$

81. If the beam be a cylinder, and r be the radius, then b is successively $2\sqrt{r^2-x^2}$, $2\sqrt{r^2-(2x)^2}$ &c. And

$$\frac{2fx^3}{rl} \times \left(0+\overline{\sqrt{r^2-x^2}}+\overline{\sqrt{r^2-x^2}+2^2\sqrt{r^2-(2x)^2}}\ \text{\&c.}\right) = W.$$

or $W = \frac{\cdot 7854\, fr^3}{l}$. (vi.)

82. If the section of the beam be an ellipse, when the strain is in the direction of the conjugate axis, we have by the same process $W = \frac{\cdot 7854\, ftc^2}{l}$ (vii.) where t is the semi-transverse, and c the semi-conjugate axis.

83. If the beam be a hollow cylinder or tube, and r be the exterior radius, $\frac{1}{n}r$ being that of the hollow part, then by the same process we find

$$W = \frac{\cdot 7854 f r^3}{l} \times \left(1 - \frac{1}{n^4}\right)^* \qquad \text{(viii.)}$$

84. If a beam be of the form shewn in *Plate* I. *Fig.* 9. (see art. 29, 30, and 31.) and d be the extreme depth, and b the extreme breadth; $q\,b$ = the difference between the breadth in the middle and the extreme breadth, and $p\,d$ the depth of the narrow part in the middle, then by the process employed in calculating Equation iv. we find,

$$W = \frac{f b d^2}{6 l} \times (1 - q p^3). \qquad \text{(ix.)}$$

85. If the middle part of the beam be entirely left out, with the exception of cross parts to prevent the upper and lower sides coming together, as in *Fig.* 11. and 12. *Plate* II. (see art. 32.) and d be the whole depth, $p\,d$ the depth of the part left out in the middle, and b the breadth, then

$$W = \frac{f b d^2}{6 l} (1 - p^3). \qquad \text{(x.)}$$

These equations shew the relation between the strength of beams, and the weight to be supported in some of the most useful cases when the load is applied, as in *Fig.* 14.; but previous to considering how

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* Dr. Young gives a rule which is essentially the same, of which I was not aware when my Principles of Carpentry was written. See Natural Philosophy, vol. ii. art. 339, B. scholium.

these equations will be affected by varying the mode of supporting the beam, it will be desirable to lay down some rules for estimating the deflexion of beams.

86. The deflexion of a beam supported as in *Fig.* 16, *Plate* II. is caused by the extension of the fibres of the upper side, and the compression of those on the under side; the neutral line ABA, retains the same length.

If we conceive the length of a beam to be divided into a great number of equal parts, and that the extension, at the upper side of the beam, of one of these parts is represented by ab, then the deflexion produced by this extension will be represented by de, and the angles acb, dce, being equal, we shall have $bc : dc :: ab : de$; the smallness of the angles rendering the deviation from strict similarity insensible.

Now, however small we may consider the parts to be, into which the length is divided, still the strain will vary in different parts of it; and consequently the deflexion; but if we consider the deflexion produced by the extension of any part, to be that which is due to an arithmetical mean between the greatest and least strains in that part, we shall then be extremely near the truth.

We have seen that the strain is as the weight and leverage directly, and as the breadth and square of the depth inversely, (see art. 77.) Our investigation will be more general by considering the weight, breadth, and depth variable, by taking l, b and d for

the length, breadth, and depth of the middle or supported point, and W for the whole weight, and 8 x, y, and w for the depth, breadth, and weight on any other point. Then, the deflexion from the strain at any point c is as

$$\frac{W\,l\,x}{2\,b\,d^2} : \frac{w \times (d\,c)^2}{y\,x^2} :: \varepsilon : \frac{2\,b\,d^2\,\varepsilon\,w\,(d\,c)^2}{W\,l\,y\,x^3}.$$

And, if z be the length of one of the parts into which we suppose the whole length divided, then the deflexion from the mean force on the length of z situate at c will be

$$\frac{2\,b\,d^2\,\varepsilon\,w\,z}{W\,l\,y\,x^3} \times \left(\frac{d\,c^2 + \overline{d\,c+z}^2}{2}.\right)$$

Since the whole deflexion DA is the sum of the deflexions of the parts, we have

$$\frac{b\,d^2\,\varepsilon\,w\,z^3}{W\,l\,y\,x^3} \times \left(\overline{0+1^2} + \overline{1^2+2^2} + \&c. \quad \overline{\overline{m-1}^2+m^2}\right) = \text{DA}.$$

(xi.)

87. Case 1. When a beam is rectangular, the depth and breadth uniform, and the load applied at one end. Then, $b=y$, $d=x$, and $W=w$. Therefore the progression becomes $\frac{\varepsilon\,z^3}{l\,d} \times (\overline{0+1^2} + \overline{1^2+2^2}$ &c.) of which the sum is $\frac{2\,\varepsilon\,l^3}{3\,d}$ = the deflexion DA*. (xii.)

88. Case 2. When the section of the beam is rectangular, and the load acts at one end, the depth being uniform, but the breadth varying as the length.

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* The same relation is otherwise determined in Dr. Young's Natural Philosophy, vol. ii. art. 325.

In this case the progression is

$$\frac{s z^2}{d} \times \overline{(0+1} + \overline{1+2} \text{ \&c.}) = \frac{s l^2}{d} = \text{the deflexion DA. (xiii.)}$$

This is the beam of uniform strength described in art. 25. *Fig*. 6.

In this case it is easily shown by other reasoning that the curve of the neutral axis is a portion of a circle, and it is well known that in an arc of very small curvature, (one of such as are formed by the deflexions of beams in practical cases,) the versed sine is sensibly proportional to the square of the sine. This will enable the reader to form an estimate of the accuracy of the method I here follow. I am perfectly satisfied that it is correct enough for use in the construction of machines or buildings; and that it is an useless refinement to embarrass the subject with intricate rules; but this explanation may be necessary to some nice theorists, who aim rather at imaginary perfection, than useful application.

89. Case 3. When the section of the beam is rectangular, the load acting at the end, the breadth uniform, and the depth varying as the square root of the length; which is the parabolic beam of equal strength. (See art. 22. *Fig*. 3. *Plate* I.)

In this case we have $\frac{s l^{\frac{1}{2}} z^{\frac{1}{2}}}{d} \times \left(\overline{0+1^{\frac{1}{2}}} + \overline{1^{\frac{1}{2}}+2^{\frac{1}{2}}} \text{ \&c.}\right)$

$= \frac{4 s l^2}{3 d} =$ the deflexion DA. (xiv.)

90. Case 4. When the section of the beam decreases from the supported point to the end where the load acts, so that the sections are similar figures,

then the curve bounding the sides of the beam will be a cubical parabola; that is, the depth will be every where proportional to the cube root of the length.

In this case the progression is

$\frac{\epsilon l^{\frac{1}{3}} z^{\frac{2}{3}}}{d} \times (\overline{0+1^{\frac{2}{3}}}+\overline{1^{\frac{2}{3}}+2^{\frac{2}{3}}}+\&c.)=\frac{6 \epsilon l^2}{5 d}$=the deflexion DA. (xv.)

91. Case 5. When a beam is of the same breadth throughout, and the vertical section is an ellipse, (see *Fig.* 8. art. 26.) the deflexion from a weight at the vertex may be exhibited in a progression as below. $\frac{l^2 \epsilon z^3}{d} \times \left\{\frac{2}{(2lz-z^2)^{\frac{3}{2}}}+\frac{8}{(4lz+4z^2)^{\frac{3}{2}}}+\&c.\ \frac{m^2}{(2lmz-m^2z^2)^{\frac{3}{2}}}\right\}$

By actually summing this progression when $m=10$, we have $\frac{\cdot 857\, l^2 \epsilon}{d}$=the deflexion DA. (xvi.)

92. Case 6. If a rectangular beam of uniform breadth and depth be loaded so that the strain upon any point c is as

$l^2\ dc \times (2l-dc) : : \mathrm{W} : w=\frac{dc \times \mathrm{W} \times (2l-dc)}{l2}$.

This value of w being substituted in Equation xi. we have

$\frac{\epsilon z^3}{d l^2}\left\{2l(\overline{0+1^2}+\overline{1^2 2^2}+\&c.)-z(\overline{0+1^3}+\overline{1^3+2^3}\&c.)\right\}=$

$\frac{\epsilon l^2}{d} \times (\tfrac{4}{3}-\tfrac{1}{2})=\frac{5 \epsilon l^2}{6 d}$=the deflexion DA*. (xvii.)

* This relation is otherwise determined by Dr. Young, Nat. Philos. vol. i. art. 329.

93. Case 7. If the section of a beam be rectangular, and the breadth uniform, but the depth at the point where the weight acts only half the depth at the point of support; then the depth at any point c will be $\frac{d}{2}\left(\frac{dc}{l}+1\right)=x$. consequently, from Equation xi. we have

$$\frac{8\,\iota\,z^3}{l\,d}\times\left(\frac{2}{(\frac{z}{e}+1)^3}+\frac{8}{(\frac{2z}{e}+1)^3}+\&c.\right)=\text{the deflexion DA,}$$

when $m=10$, the sum of the progression is nearly

$$\frac{1 \cdot 09\,\iota\,l^3}{d}=\text{DA the deflexion.} \qquad \text{(xviii.)}$$

The cases I have considered are perhaps sufficient for the ordinary purposes of business; the next object is to show how these calculations are affected by changing the position and manner of supporting the beam, or the nature of the straining force; and to compare them with experiments, and draw them into practical rules. For this purpose the most clear and the most useful plan seems to be that of taking known practical cases for illustration.

Beams supported in the Middle and strained at the Ends, as in the Beam of a Steam Engine.

94. The distance, FB, *Fig.* 14. of the direction of the straining force from the centre of motion being constantly the same, the strain will be the same in any position of the beam. (art. 77.) Also, the deflexion from its natural form will be the same in every position, because the strain is the same; and the length does not vary with the position.

Now the force acting upon the beam of a steam

engine being impulsive, the practical rules for its strength will be found in the seventh section; the formula calculated in this section being used to establish those rules.

Beams fixed at one End; as Cantilevers, Cranks, &c.

95. The strain upon a beam supported upon a fulcrum, as in *Fig.* 14. is obviously the same as when one of the ends is fixed in a wall, or other like manner; for fixing the end merely supplies the place of the weight otherwise required to balance the straining force. But though the strain upon the beam be the same, the deflexion of the point where the strain is applied will vary according to the mode of fixing the end; because the deflexion of the strained point will be that produced by the curvature of both the parts AB, and BA′.

96. Let the dotted lines in *Fig.* 17. *Plate* III. represent the natural position of a beam fixed at one end in a wall; when this beam is strained by a load at A, the compression at C will always be enough to allow the beam to curve between A′ and B: and the strain at the point A′ will obviously be the same as if a weight were suspended there that would balance the weight at A. Let ABA′ be the curvature of the beam by the load W; and $a\,\acute{a}$ a tangent to the point B. Then $A'\acute{a}$ is proportional to the deflexions produced by the strain at A′, and $A'B : BD :: A'\acute{a} :$ $Da = \frac{BD \times A\acute{a}}{A'B}$ = the deflexion from the curving of the part A′B; therefore $\frac{BD \times A'a'}{A'B} + Aa =$ the whole deflexion DA.

Now, since the deflexion is as the square of the length, (see Equation xi.—xviii. art. 87—93.) we have $(BA)^2 : (BA')^2 :: Aa : A'a' = \frac{Aa \times (BA')^2}{(BA)^2}$. Therefore, $Aa \times \left(1 + \frac{BD \times BA'}{(BA)^2}\right) = DA$.

If the angle DBA be represented by c, then $BD = BA \times \cos. c$; and putting $r = \frac{BA'}{BA}$; we have

$$Aa \times (1 + r. \cos. c) = DA.$$

But since the deflexion is always very small, in practical cases, we may always consider $\cos. c = 1$, or the radius, and then we have $Aa \times (1 + r) = DA$. (xix.)

97. In this equation r is the ratio of the length out of the wall to the length within the wall; that is $BA : BA' :: 1 : r$.

If the length of the fixed part be equal to that of the projecting part, then $r = 1$, and $2(Aa) = DA$. (xx.)

98. If the fixed part be of greater bulk than the projecting part, or it be so fixed that the extension of the fixed part would be very small, then the effect of such extension may be neglected, and the deflexion DA and Aa will be the same; particularly in the cranks of machinery, as in *Fig.* 18. because by employing this value of DA in calculating the resistance to impulsion, we err on the safe side. See Sect. 7. art. 280.

Beams supported at both Ends, as Beams for supporting Weights, &c.

99. When the same beam is supported at the ends, as in *Fig.* 19. instead of being loaded at the ends,

and supported in the middle, as in *Fig.* 14. and the inclination and sum of the load be the same in both positions, the strains will be the same.

In either position we have $W \times FB = W' \times F'B$ 8, or as $W : W' :: F'B : FB$; and therefore, (by Euclid's Elements, Prop. 18, Book 5.) $W+W' : W :: FF' : F'B$.

Consequently $W \times FB = \dfrac{\overline{W+W'} \times F'B \times FB.}{FF'}$ (xxi).

100. And the strain is as the rectangle of the segments into which the point B divides the beam; and therefore the greatest when the point B is in the middle; as has been shown by writers on mechanics*.

If the weight be applied in the middle then

$$\frac{\overline{W+W'} \times F'B \times FB}{FF'} = \frac{\overline{W+W'} \times FF'.}{4} \quad \text{(xxii)}.$$

101. When a weight is distributed over the length of a beam AB, *Fig.* 20. in any manner, the strain at any point C, may be found. For let G be the centre of gravity of that part of the load upon AC, and g that of the load upon BC. Then by the property of the lever $\dfrac{w \times AG}{AC}$ = the stress at C from the weight w of the load upon AC.

Also, $\dfrac{w' \times gB}{CB}$ = the stress at C from the weight w' of the load upon CB.

Therefore the whole stress is

$$\frac{\overline{w \times CB \times AG + w' \times AC \times gB}}{AC \times CB.}$$

* Greg. Mechan. vol. i. art. 178. cor. 2.

And by Equation xxi. art. 99. the strain will be

$$\frac{\overline{w \times CB \times AG} + \overline{w' \times AC \times gB}.}{AB} \qquad \text{(xxiii.)}$$

102. Case 1. When the weight is uniformly distributed over the length, then $AG = \frac{1}{2}AC$; and $gB = \frac{1}{2}CB$, and $w + w' = W$ the whole weight upon the beam; these values being substituted in Equation xxiii. it becomes $\frac{W \times AC \times CB}{2AB} =$ strain at C. (xxiv.)

The strain is greatest at the middle of the length, for then $AC \times CB$ is a maximum.

103. Case 2. When the load increases from A to B in proportion to the distance from A; then

$AG = \frac{2}{3}AC$, and $gB = \frac{1}{3}CB \times \frac{3AB - 2CB.}{2AB - CB}$

Also $w + w' =$ the whole weight; and $w = \frac{\frac{1}{2}AC^2 \times W}{AB}$, and

$w' = \frac{1}{2}CB \times W\frac{2AB - CB}{AB}$. These values being inserted in

Equation xxiii. it gives $\frac{W.AC}{6AB}(AB^2 - AC^2) =$ the strain at C. (xxv.)

By the principles of maxima and minima of quantities, we readily find that the strain is the greatest at the distance of $\sqrt{\frac{1}{3}}AB$ from A. And the strain will be nearly $\frac{AB^2.W}{15{\cdot}59}$ at the point of greatest strain $= \frac{WAB}{7{\cdot}75}$ when W is the whole weight.

This distribution of pressure applies to the pressure of a fluid against a vertical sheet of iron; as in lock-gates, reservoirs, sluices, cisterns, &c.

104. Case 3. When the load increases as the square of the distance from A, we find by a similar process that the strain at any point C

$$\text{is} = \frac{W.AC}{12AB^2} \times (AB^3 - AC^3). \qquad \text{(xxvi.)}$$

The point of maximum strain in this case is at the distance of $(\frac{1}{4})^{\frac{1}{3}}AB$ from A.

Practical Rules and Examples.

Resistance to Cross Strains.

105. Prop. I. To determine a rule for the breadth and depth of a beam, to support a given weight or pressure, when the distance between the supported or strained points is given; when the breadth and depth are both uniformly the same throughout the length, and the strain does not exceed the elastic force of cast iron.

106. Case 1. When a beam is supported at the ends and loaded in the middle, as in *Fig.* 19. From Equation xxii. art. 100. and iv. art. 79. taking W for the weight, we have $\frac{W\,l}{4} = \frac{f b d^2}{6}$ where l=FF′; *Fig.* 19. and the value of f is the only part required from experiment; and $\frac{3\,l\,W}{2\,b\,d^2} = f$. Now in the experiment described in art. 45. Sect. 5. the bar returned to its natural state when the load was 300 lbs. and I was perfectly satisfied that it would bear more than that weight without destroying its elastic force. Therefore, from this experiment $\frac{3 \times 34 \times 300}{2} = f = 15{,}300$ lbs.

That is, cast iron of the quality described in art. 45. will bear 15,300lbs. upon a square inch, when drawn in the direction of its length, without producing permanent alteration in its structure. If this value of f be employed, our equation becomes $\frac{3\,l\,W}{2\times 15300} = b\,d^2$; or, as it is convenient to take l in feet, $\frac{3\times 12\times l\times W}{2\times 15300} = \frac{l\,W}{850} = b\,d^2$.

107. Rule 1. To find the breadth of an uniform cast iron beam, to bear a given weight in the middle.

Multiply the length of bearing in feet, by the weight to be supported in pounds; and divide this product by 850 times the square of the depth in inches; the quotient will be the breadth in inches required.

108. Rule 2. To find the depth of an uniform cast-iron beam, to bear a given weight in the middle.

Multiply the length of bearing in feet, by the weight to be supported in pounds, and divide this product by 850 times the breadth in inches; and the square root of the quotient will be the depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the nth part of the depth, n representing any number at will. Then the rule becomes—

109. Rule 3. Multiply n times the length in feet, by the weight in pounds; divide this product by

850, and the cube root of the quotient will be the depth required: and the breadth will be the nth part of the depth.

It may be remarked here, that the rules are the same for inclined as for horizontal beams, when the horizontal distance FF′ *Fig.* 19. is taken for the length of bearing.

110. Example 1. In a situation where the flexure of a beam is not a material defect, I wish to support a load which cannot exceed 33,600 lbs. or 15 tons, in the middle of a cast-iron beam, the distance of the supports being 20 feet; and making the breadth a fourth part of the depth.

In this case $n=4$ and $\frac{4\times 20\times 33,600}{850}=3162{\cdot}35$.

The cube root of 3162·35 is nearly 14·68 inches the depth required; the breadth is $\frac{14{\cdot}68}{4}=3{\cdot}87$ inches.

In practice therefore I would use whole numbers, and make the beam 15 inches deep, and 4 inches in breadth.

111. Case 2. When the beam is supported at the ends, but the load is not in the middle between the supports. In this case $\frac{W.FB\times F'B}{l}=\frac{f\,b\,d^2}{6}$ (Equation xxi. art. 99. and iv. art. 79.) consequently $\frac{4FB\times F'B\times W}{850\,l}=bd^2$.

112. Rule. Multiply the distance FB in feet (see *Fig.* 19.) by the distance F′B in feet, and 4 times this product divided by the whole length FF′ in feet, will

give the effective leverage of the load, which being used instead of the length in any of the rules to Case 1. Prob. 1. the breadth and depth may be found by them.

113. Example. Taking the same example as the last, except that instead of placing the 15 tons in the middle, it is to be applied at 5 feet from one end; therefore we have FB=5 feet, and consequently F′B=15 feet; and $\frac{5\times15\times4}{20}=15$ the number to be employed instead of the whole length in Rule 3. That is $\frac{4\times15\times33{,}600}{850}=2{,}372$ nearly; and the cube root of 2,372 is nearly 13·34 inches the depth for the beam, and $\frac{13\cdot34}{4}=3\cdot33$ inches for the breadth, or nearly 13⅓ inches by 3⅓ inches. In the former case, it was 15 inches by 4 inches.

114. Case 3. When the load is uniformly distributed over the length of the beam, which is supported at both ends.

In this case $\frac{W\,l}{8}=\frac{f\,b\,d^2}{6}$; (see Equation iv. art. 79. and xxiv. art. 102.) hence $\frac{l\,W}{2\times850}=b\,d^2$; and the same rules apply as in Case 1. art. 107, 108, and 109, by changing the divisor from 850 to 1700.

115. Example. In a situation where I cannot make use of an arch for want of abutments, it is necessary to leave an opening fifteen feet wide, in an eighteen inch brick wall; required the depth of two cast-iron beams to support the wall over the opening,

each beam to be two inches thick, and the height of the wall intended to rest upon the beam being thirty feet?

The wall contains $30 \times 15 \times 1\frac{1}{2} = 675$ cubic feet; and as a cubic foot of brick-work weighs about 100 lbs. the weight of the wall will be about 67,500 lbs.; and half this weight, or 33,750 lbs. will be the load upon one of the beams. Since the breadth is supposed to be given, the depth will be found by Rule 2. art. 108, if 1700 be used as the constant divisor; thus $\frac{15 \times 33.750}{1700 \times 2} = 149$ nearly. The square root of 149 is $12\frac{1}{4}$ nearly, therefore each beam should be $12\frac{1}{4}$ inches deep, and 2 inches in thickness. This operation gives the actual strength necessary to support the wall, but I have usually taken double the calculated weight in practice, to allow for accidents.

116. Case 4. When a beam is fixed at one end, and the load applied at the other; also when a beam is supported upon a centre of motion. By Equation iv. art. 79. $W\,l = \frac{f\,b\,d^2}{6}$; and taking l in feet, and $f = 15{,}300$ lbs. we obtain $\frac{W\,l}{212 \cdot 5} = b\,d^2$, but the divisor 212 will be always sufficiently near for practice.

117. Rule 1. In a beam, fixed at one end, take BD for the length, *Fig.* 17. *Plate* III. or if the beam be supported in the middle, as in *Fig.* 14. *Plate* II. take BF or BF′ for the length, observing to use the weight which is to act on that end in the calculation. Then calculate the strength by the rules to Case 1.

art. 107, 108, 109, using 212 instead of 850 as a divisor.

118. Rule 9. If the weight be uniformly distributed over the length of the beam, employ 425 as a divisor, instead of 850 in the rules to Case 1. art. 107, 108, 109.

119. Example. Required the depth for the cantilevers of a balcony to project 4 feet, and to be placed 5 feet apart, the weight of the stone part being 1000 lbs. the breadth of each cantilever 2 inches, and the greatest possible load that can be collected upon 5 feet in length of the balcony 2200 lbs.

Here the weight is $1000+2200=3200$ lbs. and by Rule 2. Case 4. $\frac{3200\times 4}{2\times 425}=15{\cdot}1$ nearly, and the square root of $15{\cdot}1$ is $3{\cdot}89$ nearly, the depth required.

120. *Remark.* The depth thus determined should be the depth at the wall as AB, *Fig.* 21. *Plate* III. and if the breadth be the same throughout the length, the cantilever will be equally strong in every part, if the under side be bounded by the straight line BC*; therefore, whatever ornamental form may be given to it, it should not be reduced in any part to a less depth than is shown by that line.

121. The rules of this case apply to the teeth of wheel-work, where the length is the length of the teeth, and the depth is the thickness of the teeth.

Example. Let the greatest power acting at the pitch line of the wheel be 6000 lbs. and the thickness

* Emerson's Mechanics, 4to. edit. Prop. 73. Cor. 2.

of the teeth $1\frac{1}{2}$ inch, the length of the teeth being 0·25 feet; it is required to determine the breadth of the teeth?

By Rule 1. art. 117. $\frac{6000 \times 0\cdot25}{212 \times \cdot5^2} = \frac{500}{477} = 3\cdot2$ inches, the breadth required.

Now in order that these teeth may be capable of offering a sufficient resistance after being worn down by friction, which takes place in a greater or less degree in all wheel-work, the breadth thus calculated should be at least doubled; therefore in the above given example the breadth should be $6\frac{1}{2}$ inches.

122. Case 5. When the pressure upon a beam increases as the distance from one of its points of support. Since the point of greatest strain is at $\sqrt{\frac{1}{3}}\,l$ from the point A where the strain begins at, (see *Fig.* 20.) we have by art. 103. and 79. $\frac{W\,l}{7\cdot75} = \frac{f\,b\,d^2}{6}$, or when l is in feet, and $f = 15{,}300$ lbs.; $\frac{W\,l}{1{,}647} = b\,d^2$; a result which differs so little from Case 3. that the same rule may serve for both cases.

123. Prop. 2. To determine a rule for the diagonal of a uniform square beam to support a given strain in the direction of that diagonal; when the strain does not exceed the elastic force of cast iron.

124. Case 1. When the beam is supported at the ends and loaded in the middle, $\frac{W\,l}{4} = \frac{f\,a^3}{24}$; art. 100. and 80. or when l is in feet, and $f = 15{,}300$ lbs. $\left(\frac{W\,l}{212}\right)^{\frac{1}{3}} = a.$

125. Rule. Multiply the length in feet by the weight in pounds, and divide the product by 212; the cube root of the quotient is the diagonal of the beam in inches.

126. Case 1. When a beam is supported at the ends, and the strain is not in the middle of the length. $\frac{W \times FB \times F'B}{l} = \frac{f a^3}{24}$, art. 80. and 99. or when $f =$ 15,300 lbs. and the length and distances FB, F'B from the ends are in feet, $(\frac{W \times FB \times F'B}{53 l})^{\frac{1}{3}} = a$.

127. Rule. Multiply the weight in pounds by the distance FB in feet, and multiply this product by the distance from the other end, or F'B in feet. (see *Fig.* 19.) Divide the last product by 53 times the length, and the cube root of the quotient will be the diagonal of the beam in inches.

I limit the rules to these cases only, because a beam is seldom placed in the position described in this proposition. Examples are omitted for the same reason.

128. Prop. 3. To determine a rule to find the diameter of a solid cylinder, to support a given strain, when the strain does not exceed the elastic force of cast iron.

If the diameter be not uniform, the diameter determined by the rule will be that at the point of greatest strain, and the diameter at any other point should never be less than corresponds to the form of equal strength.

129. Case 1. When a solid cylinder is supported at

the ends, and the weight acts at the middle of the length. $\frac{W\,l}{4}{\cdot}7854fr^3$, art. 81. and 100. or when l is in feet, $f=15{,}300$ lbs. and $d=$ the diameter $=2\,r$; we have $\left(\frac{W\,l}{500}\right)^{\frac{1}{3}}=d$.

130. Rule. Multiply the weight in pounds by the length in feet; divide this product by 500, and the cube root of the quotient will be the diameter in inches.

The figure of equal strength for a solid of which the cross section is every where circular, is that generated by two cubic parabolas, set base to base*, the bases joining at the section where the strain is the greatest.

131. Example. Required the diameter of a horizontal shaft of cast iron to sustain a pressure of 2000 lbs. in the middle of its length; the length being 12 feet? In this case we have $\frac{2000\times 20}{500}=80$; and the cube root of 80 is 4·31 inches nearly, which is the diameter required.

This is supposed to be a case where the flexure is of no importance, otherwise the diameter must be determined by the rules for flexure.

132. Case 2. When a cylinder is supported at the ends, but the strain is not in the middle of the length. By art. 81. and 99. $\frac{W\times FB\times FB}{l}={\cdot}7854fr^3$; or when

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* Emerson's Mechanics, 4to. edit. Prop. 73, Cor. 4.

the lengths are in feet, d is the diameter, and $f=$ 15,300; the equation becomes $\left(\frac{4W \times FB \times FB}{500\, l}\right)^{\frac{1}{3}} = d.$

133. Rule. Multiply the rectangle of the segments, into which the strained point divides the beam, in feet, by 4 times the weight in pounds; when this product is divided by 500 times the length in feet, the cube root of the quotient will be the diameter of the cylinder in inches.

The figure of equal strength is the same as in Case 1. art. 30.

134. Example. Required the diameter of a shaft of cast iron, to resist a pressure of 4000 lbs. at three feet from the end, the whole length of the shaft being 14 feet. In this example $\frac{3 \times 11 \times 4 \times 4000}{500 \times 14} =$ 75·43. The cube root of 75·43 is nearly 4·23 inches, the diameter required.

135. Case 3. When a load is uniformly distributed over the length of a solid cylinder supported at the ends only. By art. 81. and 102. $\frac{W\, l}{8} = \cdot 7854 f r^3$;

therefore, when l is in feet, d the diameter, and $f=$ 15,300, we have $\left(\frac{W\, l}{1000}\right)^{\frac{1}{3}} = d = \frac{1}{10}\,(W\, l)^{\frac{1}{3}}.$

136. Rule. Multiply the length in feet, by the weight in pounds, and one-tenth of the cube root of the product will be the diameter in inches.

The figure of equal strength for a uniform load, the section being every where circular, is that gene-

rated by the revolution of a curve of which the equation is $a\ (lx-x^2)^{\frac{1}{2}}=y$*.

137. Example. A load of 6 tons (or 13,440 lbs.) is to be uniformly distributed over the length of a solid cylinder of cast iron, of which the length is 12 feet; required its diameter, so that the load shall not exceed its elastic force?

In this case $12\times 13{,}440=161{,}280$; and the cube root of 161,280 is 54·44, and 1-10th of this is 5·444 inches, the diameter required.

138. Case 4. When a cylinder is fixed at one end and the load applied at the other; also when a cylinder is supported on a centre of motion. By art. 81. $W\,l=\cdot 7854 fr^3$. therefore, when d is the diameter, l is in feet, and $f=15{,}300$ lbs. we have

$\left(\frac{8Wl}{1000}\right)^{\frac{1}{3}}=d.$ or $\frac{1}{5}(W\,l)^{\frac{1}{3}}=d.$

The figure of equal strength is the same as in Case 1. art. 130.

139. Rule. Multiply the leverage the weight acts with, in feet, by the weight in pounds; *the fifth part* of the cube root of this product will be the diameter required in inches.

140. Prop. 4. To determine a rule for the exterior diameter of an uniform tube or hollow cylinder † to

* Emerson's Mechanics, Prop. 73. Cor. 3.

† A considerable accession of strength and stiffness is gained by making shafts hollow; but it is difficult to get them cast sound, therefore shafts of this kind require to be carefully proved.

resist a given force, where the strain does not exceed the elastic force of cast iron.

141. Case 1. When a tube is supported at the ends, and the load acts at the middle of the length. By art. 83. and 100. $\frac{Wl}{4} = \cdot 7854 f r^3 (1 - \frac{1}{n^4})$; hence when d is the diameter, l the length in feet, and $f = 15{,}300$ lbs. we have $\left(\frac{Wl}{500(1 - \frac{1}{n^4})}\right)^{\frac{1}{3}} = d.$

142. Rule. Fix on some proportion between the diameters; so that the exterior diameter is to the interior diameter as 1 is to N; the number N will always be a decimal, and ought not to exceed 0 8*.

Then multiply the length in feet by the weight to be supported in pounds. Also, multiply 500 by the difference between 1 and the fourth power of N. and divide the product of the length and weight by the last product, and the cube root of the quotient will be the diameter in inches.

The interior diameter will be N × the exterior diameter, and half the difference of the diameters will be the thickness of metal.

143. Example. Let the weight of a water wheel, including the weight of the water in the buckets, be

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* In a large shaft there should be a tolerable bulk of metal to secure a perfect casting. Mr. Buchanan, in his "Essay on the Shafts of Mills," p. xxxiv. describes a hollow shaft of which the exterior diameter was 16 inches, and the interior one 12 inches, therefore $16 : 12 :: 1 : N = \frac{12}{16} = \cdot 75$. This shaft was for an overshot water-wheel of 16 feet diameter

44,800 lbs. and the whole length of the shaft 8 feet; from which deducting 5 feet, the width of the wheel leaves 3 feet for the length of bearing: required the diameter of a hollow shaft for it?

Making N=·7, its fourth power is ·343 and 1−·343=·657. Therefore by the rule we have $\frac{3 \times 44800}{500 \times \cdot 657} = 409$; and the cube root of 409 is nearly 7·5 inches the exterior diameter, and 7·5×·7=5.25 inches the interior diameter.

144. Case 2. When a tube is supported at the ends, but the strain is not in the middle of the length. When the necessary substitutions are made, we have, by art. 83. and 99. $\left(\frac{4W \times FB \times F'B}{500\, l \times (1-N^4)}\right)^{\frac{1}{3}} = d.$

145. Rule. Multiply the rectangle of the segments into which the strained point divides the beam, in feet, by four times the weight in pounds; call this the first product.

Multiply 500 times the length, in feet, by the difference between 1 and the fourth power of *N*; (*N* being the interior diameter when the exterior diameter is unity;) call this the second product.

Divide the first product by the second, and the cube root of the quotient will be the exterior diameter of the tube in inches.

146. Example. Let the weight of a wheel and other pressure upon a shaft be equal to 36,000 lbs. the distance of the point of stress from the bearing at one end being 3 feet, and the distance from the

other bearing 1·5 feet, N being ·8; required the exterior and interior diameter of the shaft?

The fourth power of ·8 is ·409, and $1-·409=·591$. Therefore by the rule $\frac{31\times1·5\times4\times36000}{500\times4·5\times·591}=485$; and the cube root of 485 is 7·86 inches the exterior diameter, and $7·86\times·8=6·3$ inches the interior diameter.

Cases 3. and 4. are not likely to occur in the practical application of tubes, but they may be supplied by Case 3. and 4. for solid cylinders, by dividing the diameter of the solid cylinder by the cube root of the difference between 1 and the fourth power of N.

147. Prop. 5. To determine a rule for finding the depth of a beam of the form of section shown in *Fig.* 9. *Plate.* I. to resist a given force when the strain does not exceed the elastic force of cast iron.

148. Case 1. When the beam is supported at the ends, and the load acts in the middle of the length. By art. 84. and 100. $\frac{Wl}{4}=\frac{fbd^2}{6}(1-qp^3)$; or making l=the length in feet, and $f=15,300$ lbs.

$\frac{Wl}{850}=bd^2(1-qp^3)$*.

149. Rule. Assume a breadth *ab* *Fig.* 9. that will answer for the purpose the beam is intended for;

* If we make $p=·7$, and $q=·6$; then, $850\ (1-qp^3)=675$; and the rule is $\frac{Wl}{675}=bd^2$; and the breadth of the middle part $=·4\,b$, and the depth of the middle part $·7\,d$.

and let this breadth multiplied by some decimal q be equal to the sum of the projecting parts, or, which is the same thing, equal to the difference between the breadth of the middle part and the whole breadth.

Also, let p be some decimal which multiplied by the whole depth will give the depth of the middle or thinner part ef in the figure.

Multiply the length in feet by the weight in pounds, and divide this product by 850 times the breadth multiplied into the difference between unity and the cube of p multiplied by q; the square root of the quotient will be the depth in inches.

The figure of equal strength for this case is formed by two common parabolas put base to base, as shown by the dotted lines in *Fig.* 22.; for $l : d^2$ a property of the parabola, the other being constant quantities. *Fig.* 22. shows how it may be modified to answer in practice. When a figure of equal strength is used, the depth determined by the rule is that at the point of greatest strain as CD in the figure.

150. Example 1. Required the depth of a beam of cast iron of the form of section shown in *Fig.* 9. *Plate* I. to bear a load of 33,600 lbs. in the middle of the length, the length being 20 feet, and the breadth $a\,b$, 3 inches.

Take ·625 for the decimal q, and ·7 for the decimal p, which are proportions that will be found to answer very well in practice. Then

$$\frac{20 \times 33600}{850 \times 3 \times (1 \cdot 625 \times \cdot 7^3)} = \frac{20 \times 33600}{850 \times 3 \times \cdot 7856} = 335.4 \text{ nearly,}$$

and the square root of 335·4 is 18·4 inches, the depth required.

The depth bd being 18·4, the depth ef will by $18·4 \times ·7 = 12·88$ inches; also $3 \times ·625 = 1·875$, and $3 - 1·875 = 1·125$ inches, the breadth of the middle part of the section.

Comparing this with the example art. 110. it will be found that the same weight requires only about two-thirds of the quantity of iron to support it, when the beam is formed in this manner.

Example 2. The same rule applies to determining the size of the rails for an iron railway, where economy with strength and durability is of much importance. As the weight has to move over the length of the rail, the figure of equal strength is that shown in *Plate* III. *Fig.* 24. only it should be placed with the straight side upwards.

Suppose the weight of a coal waggon to be about four tons, 8960 lbs. but the utmost strain on a rail cannot exceed half this weight or 4480 lbs., which will be allowing half the strength nearly for accidents. The usual length of one rail is three feet*,

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* It is worthy of consideration whether this be the most economical length, or not, for rails. This may be done as follows: the weight of a bar of iron, an inch square and 700 feet long, is one ton; therefore, for a length of 700 feet the area of the bar in inches multiplied by the price of a ton of iron will be the amount of 700 feet of rail. Make $\frac{700}{x}$ the length of a single rail; then, supposing the rail all of the same thickness, $\sqrt{\frac{W \times 700}{850 \times bx}} = d$, the depth, and

and supposing the breadth to be two inches, then, by the manner of calculation shown in the note to art. 148. $\frac{Wl}{675 \times b} = \frac{4{,}480 \times 3}{675 \times 2} = d^2 = 9{\cdot}96$; and the square root of $9{\cdot}96 = 3{\cdot}16$ inches, the depth in the middle of the length. Also, $3{\cdot}16 \times {\cdot}7 = 2{\cdot}212$ inches, the depth of the thin part in the section at the middle of the length, and $2 \times {\cdot}1 = 0{\cdot}8$ inches, the thickness of the middle part of the section.

The depth of a rail, all of the same thickness, would be 2·83 inches in the middle, calculated by Rule 2. art. 108.

151. Case 2. When the beam is supported at the ends, but the load not applied in the middle between the supports. When l is the length in feet, and $f = 15{,}300$ lbs. $\frac{4 FB \times FB \times W}{850 l (1 - p^2 q)} = b d^2$ by art. 84. and 99.

152. Rule. Multiply the rectangle of the segments into which the strained point divides the

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when it is reduced at the ends, ${\cdot}7 b \sqrt{\frac{W \times 700}{850 \times bx}} =$ the area, and calling A the price of a ton of iron, and B the price of fixing, and materials for one block; then the price of 700 feet will be ${\cdot}7 A b \sqrt{\frac{W \times 700}{850 bx}} + x B$; $= \frac{{\cdot}64 A \sqrt{Wb}}{\sqrt{x}} + B x$.

Hence by the rules of maxima and minima it appears that the price will be the least when the numbers of supports for 700 feet is $= \left(\frac{{\cdot}32 A \sqrt{Wb}}{B}\right)^{\frac{2}{3}}$; wherein W is half the weight of a waggon and its load, in pounds.

beam in feet by 4, and divide this product by the length in feet; use this quotient instead of the length of the beam, and proceed by the last rule.

153. Example. Let the load to be supported be 33,600 lbs. at 5 feet from one end, the whole length being 20 feet. Also let the breadth of the widest part *a b* *Fig.* 9. be 4 inches.

Here FB=5 feet, therefore F'B=15 feet, and $\frac{4\times5\times15}{20}=15$: the multiplier to be used instead of the whole length in the rule.

Let $p=\cdot7$, and $q=\cdot625$; then $\frac{15\times33600}{850\times4\times(1-.625\times7^3)}=$ 189 nearly, of which the square root is 13·5 inches, the depth required.

The depth *ef* will be $\cdot7\times13\cdot5=9\cdot45$ inches, and the breadth of the middle part of the section will be $4-4\times\cdot625=4-2\cdot5=1\cdot5$ inches.

154. Case 3. When the load is uniformly distributed over the length of a beam. In this case $\frac{W\,l}{1700\,(1-q\,p^3)}=b\,d^2$ by art. 84. and 102.

155. Rule. Use 1700 instead of 850 as a divisor, and proceed by the rule to Case 1. art. 149.

The form of equal strength for this case, when the breadth is uniform, is an ellipse, but in practical cases it will require to be altered to the form shown in *Fig.* 24.

156. Example. It is proposed to form a fire-proof room, but from its situation it cannot be vaulted in the ordinary way on account of the strong abut-

ments required for common vaulting, and also common vaulting is objectionable because so much space is lost in a low room. The shortest direction across the room is 12 feet, and the depth for the floor is 12 inches, and the beam 10 inches, and if iron beams be laid across at 3 feet apart, and arched between with 9 inch brick arches, it is required to find the breadth for the beams. See *Fig.* 10. *Plate* I.

The cube quantity of brick work resting upon one joist will be $12 \times 3 \times \cdot 75 = 27$ cubic feet; and the weight of a cubic foot being nearly 100 lbs. the weight of the brick work will be 2700 lbs.

But since the space above is to be used, and the greatest probable extraneous weight that will be in the room will arise from its being filled with people, we may take that weight at 120 lbs. per superficial foot, or $12 \times 3 \times 120 = 4320$ lbs.

Therefore the whole load may probably amount to $2700 + 4320 = 7020$ lbs. on each beam; which may be estimated at 8500 lbs. to include the weight of beams and flooring over them.

Making $q = \cdot 625$, and $p = \cdot 7$, we have

$$\frac{12 \times 8500}{1700 \times 10^2 \times (1 - \cdot 625 \times \cdot 7^3)} = \frac{6}{7 \cdot 856} = \cdot 77$$ nearly the breadth required in inches.

Therefore $\cdot 77 \times \cdot 625 = \cdot 481$ and $\cdot 77 - \cdot 481 = \cdot 289$ the breadth of the middle part; also $10 \times \cdot 7 = 7$ inches the depth of the middle part.

Otherwise:—Fix upon 2 inches for the breadth; then by the rule we have $\dfrac{12 \times 8500}{1700 \times 2 \times (1 - \cdot 625 \times 7^3)} =$

49·4, of which the square root is a little more than 7 inches, the depth required. And $7 \times \cdot 7 = 4\cdot 9$ inches the depth of the middle part, and $2 - 2 \times \cdot 625 = 2 - 125 = \cdot 75$ the breadth of the middle part.

By fixing the breadth you avoid the risk of calculating for a thinner beam than is sufficient to support firmly the abutting course of bricks.

157. Case 4. When a beam is fixed at one end, and the load applied at the other. Also, when a beam is supported upon a centre of motion. By art. 84. $Wl = \frac{fbd^2}{6} \times (1 - p^3q)$, or when l is in feet, and $f = 15{,}300$ lbs. $\frac{Wl}{212(1-qp^3)} = bd^2$.

158. Rule 1. Calculate by the rule to Case 1. art. 149. using 212 instead of 850 for a divisor.

The figure of equal strength is a parabola; see *Fig.* 25. and 26.

159. Rule 2. If the weight be uniformly distributed over the length, take the whole load upon the beam for the weight, and calculate by the rule to Case 1. art. 149. except using 425 instead of 850 as a divisor.

160. Prop. 6. To determine a rule for finding the depth of a beam when part of the middle is left open, as in *Fig.* 11. 12. and 27. so that it will resist a given force; the strain not exceeding the elastic force of the material.

162. When the depth is more then 12 or 14 inches, angular parts in the middle become necessary as in *Fig.* 27. the disposition of the middle part may

in a great measure be regulated by fancy, provided it allows of sufficient diagonal and cross ties to bind the upper and lower part together. The middle parts should be made of the same size as the other, in order that they may not be rendered useless by irregular contraction.

If the beams be required so long as not to be made in a single casting, and it is not a good plan to cast in very long lengths, then they may be joined in the middle, as in *Fig.* 27. The connection is made at the lower side only; at the upper side let the parts abut against one another, with only some contrivance to steady them while they become fixed in their places and loaded. *Fig.* 28. is a plan of the under side showing how the connection may be made.

162. Case 1. When the beam is supported at the ends, and the load acts at the middle of the length. By art. 85. and 100. $\frac{Wl}{4} = \frac{fbd^2}{6}(1-p^3)$, or making l = the length in feet, and $f = 15{,}300$ lbs.;

$\frac{Wl}{850(1-p^3)} = bd^2$, or when $p = \cdot 7$, $\frac{Wl}{558} = bd^2$.

163. Rule. Multiply the length in feet, by the weight to be supported in pounds; and divide this product by 558 times the breadth in inches; the square root of the quotient will be the depth required in inches. Consult art. 32. and 34. respecting the form of beams of this kind. The depth between the upper and lower part of the beam will be $\cdot 7\,d$ inches, where d is the depth found by the rule.

164. Example. A beam for a 30 feet bearing is intended to sustain a load of 6 tons, (13,440 lbs.) in the middle, the breadth to be 4 inches; required the depth?

By the rule $\frac{30 \times 13440}{4 \times 558} = 180 \cdot 556$; the square root of $180 \cdot 556$ is $13 \cdot 44$ inches, the whole depth.

The depth between the upper and lower part is $\cdot 7 \times 13 \cdot 44 = 9 \cdot 408$ inches. If the depth be given, suppose 16 inches, and the breadth be required, then $\frac{30 \times 13440}{16 \times 16 \times 558} = 2 \cdot 822$, the breadth in inches; when the depth is 16 inches, and the depth between the upper and lower parts is $\cdot 7 \times 16 = 11 \cdot 2$ inches.

165. Case 2. When a beam is supported at the ends, but the load is not applied at the middle.

When l is the length in feet, $p = \cdot 7$, and $f =$ 15,300 lbs. $\frac{4 \mathrm{BC} \times \mathrm{CD} \times \mathrm{W}}{558\, l} = bd^2$; (see *Fig.* 12. *Plate* I.) or $\frac{\mathrm{BC} \times \mathrm{CD} \times \mathrm{W}}{139\, l} = b\, d^2$.

166. Rule. Multiply the rectangle of the segments into which the strained point divides the beam, in feet, by the weight in pounds, and divide this product by 139 times the length in feet multiplied by the breadth in inches; the square root of the quotient will be the depth required in inches.

The depth between the upper and lower side will be $\cdot 7 \times$ by the whole depth. Consult art. 32. and 34. respecting the form, &c. of beams of this kind.

167. Example. Let CB *Fig.* 12. be 10 feet, and

DC 6 feet; and therefore BD the length 16 feet: and the weight to be supported, at A, 20,000 lbs. the breadth of the beam being 2 inches; required the depth?

By the rule $\frac{10 \times 6 \times 20000}{139 \times 16 \times 2} = 270$, and the square root of 270 is $16\frac{1}{2}$ inches nearly.

Also $\cdot 7 \times 16\ 5 = 11{\cdot}55$ inches=the depth from a to b in *Fig.* 12.

168. Case 3. When a load is distributed uniformly over the length of a beam. When the length is in feet, $p = \cdot 7$, and $f = 15{,}300$ lbs. $\frac{W\,l}{1116} = b\,d^2$. by art. 85. and 102.

169. Rule. Multiply the whole weight in pounds by the length in feet; divide this product by 1116 times the breadth in inches, and the square root of the quotient will be the depth in inches.

Multiply this depth by $\cdot 7$, which will give the depth between the upper and lower parts. Respecting the form of the beam, see art. 32.

170. Example. It is required to support a wall, 20 feet in height, and 18 inches in thickness, over an opening 26 feet wide, by means of two beams of cast iron, each 3 inches in thickness; required the depth?

Suppose a cubic foot of brick work to weigh 100 lbs.; then $20 \times 1{\cdot}5 \times 26 \times 100 = 78{,}000$ lbs. the weight of the wall.

Therefore by the rule $\frac{78{,}000 \times 26}{1116 \times 6} = 303$ nearly, and

the square root of 303 is $17\frac{1}{2}$ inches, the depth required.

The depth between the upper and lower part is $\cdot 7 \times 17 \cdot 5 = 12 \cdot 25$ inches.

171. Case 4. When a beam is fixed at one end, and the load is applied at the other. Also, when the load acts at one end of a beam supported on a centre of motion. By art 85. we have, when the length is in feet, $p = \cdot 7$, and $f = 15{,}300$ lbs. $\frac{W\,l}{139} = b\,d^2$.

172. Rule. Calculate by the rule to Case 1. art. 163. using 139 instead of 558 for a divisor.

If the weight be uniformly distributed over the length of a beam fixed at one end, divide the weight by 2, and proceed as above directed.

Deflexion from Cross Strains.

173. Prop. 7. To determine a rule for finding the deflexion of a cast-iron beam, when the section is rectangular, and uniform throughout the length; the strain being 15,300 lbs. upon a square inch.

The same rules will apply to solid and hollow cylinders, to beams formed as *Fig.* 9. 11. 12. and 26. when they are uniform throughout their length, and the depth used as a divisor is the greatest depth.

174. Case 1. When a beam is supported at the ends, and loaded in the middle, as in *Fig.* 1.

By art. 87. $\frac{2\,\iota\,l^2}{3\,d} =$ the deflexion, when l=half the length; therefore, $\frac{3\,d \times DA}{2\,l^2} = \iota$ = the greatest exten-

sion of an inch length while the elastic force remains perfect. According to the experiment described in art. 45. the elastic force was perfect when the bar was loaded with 300lbs. hence we have $\frac{3\,d \times DA}{2\,l^2} = \frac{3 \times 1 \times \cdot 16}{2 \times 17^2} = \frac{1}{1204} = \cdot 00083$ inches $= \epsilon$ the extension of an inch in length, by a force equal to 15,300 lbs. upon a square inch; or generally, cast iron is extended $\frac{1}{1204}$th part of its length by a force equal to 15,300 lbs. upon a square inch.

If this value of ϵ be substituted in the equation, and l be made the whole length in feet, we have $\frac{2 \times \cdot 00083 \times 12^2 \times l^2}{3 \times 4 \times d} = DA$, or $\frac{\cdot 01992\,l^2}{d} = DA$; hence it appears that the equation $\frac{\cdot 02\,l^2}{d} = DA$ may be used without sensible error.

Consequently, the deflexion of an uniform rectangular beam supported at the ends, may be determined by the following rule:

175. Rule. Multiply the square of the length in feet by $\cdot 02$; and this product divided by the depth in inches, is equal to the deflexion in inches.

176. Example. Required the deflexion in the middle of a beam 20 feet long, and 15 inches deep, when strained to the extent of its elastic force?

By the rule $\frac{\cdot 02 \times 20^2}{15} = \cdot 533$ inches; therefore a beam loaded as in example, art. 110. will bend more

than half an inch in the middle. If it be wished to reduce it to a quarter of an inch, double the breadth.

The deflexion of an uniform beam may also be found by Table II. art. 6.

177. Case 2. When an uniform rectangular beam is supported at the ends, and the load is equally distributed over the length. It has been shown in art. 102. Equation xxiv. that in this case the strain at any point is as the rectangle of the segments into which that point divides the beam; and the deflexion for that case is calculated by art. 92. Equation xvii. And by comparing Equation xii. and xvii. $\frac{2}{3} : \frac{5}{6} :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 025\, l^2}{d}$. Therefore the deflexion DA in the middle of a beam uniformly loaded is $= \frac{\cdot 025\, l^2}{d}$.

178. Rule. Multiply the square of the length in feet by ·025; and the quotient from dividing this product by the depth in inches, will be the deflexion in the middle in inches.

179. Example. Let it be required to determine the deflexion that may be expected to take place in the example to Case 3. Prop. 1. art. 115. where the length is 15 feet and the depth 12¼ inches?

By the rule $\frac{15 \times 15 \times \cdot 025}{12 \cdot 25} = \cdot 46$ inches the deflexion required.

180. This mode of calculation may often remove groundless alarm, as well as inform us when a structure is dangerous; for if a beam be loaded so as to

bend more than is determined by the rule which applies to it, the structure may be justly deemed insecure. We also, by this mode of calculation, have an easy method of trying the goodness of a beam; for let it be loaded with any part, as for example 1-4th of the weight it should bear, then the deflexion ought to be 1-4th of the calculated deflexion. When a beam is tried by loading it with more than the weight it is intended to bear, it may be so strained as to break with the lesser weight, besides the difficulty and danger in trying such an experiment.

181. Case 3. When a beam is fixed upon a centre of motion, and the force applied at the other end, the flexure of the fixed part being insensible. The cranks of engines are in this case.

The flexure will be the same as in Case 1. art. 174. but the length of the beam being only half the length in that case, we have $\frac{\cdot 08\, l^2}{d} = \mathrm{DA}$ the deflexion.

182. Case 4. If an uniform rectangular beam be fixed at one end, and the force be applied at the other, the deflexion of the end where the force is applied will be $\frac{\cdot 08 l^2}{d} \times (1+r)$.

For the deflexion from the extension of the projecting part of the beam is $\frac{\cdot 08\, l^2}{d}$, where l is the length of that part in feet; and if r be equal the $\frac{\text{length of fixed part,}}{l}$ then by Equation xix. art. 96. $\frac{\cdot 08\, l^2}{d} \times (1+r)$ = the deflexion.

183. Rule. Divide the length of the fixed part of the beam by the length of the part which yields to the force, and add 1 to the quotient; then multiply the square of the length in feet by the quotient so increased, and also by ·08; this product divided by the depth in inches will give the deflexion in inches.

184. Example. Conceive a beam AB *Fig.* 26. to be uniform, and to be the beam of a pumping engine, the end B working the pumps, and the end A where the power acts 10 feet from the centre of motion, the end B 7 feet from the centre of motion, and the strain at B equal to the elastic force of the beam; through how much space will the point A move before the beam transmits the whole power to B, the depth of the beam being 12 inches?

In this case $\frac{7}{10} = \cdot 7$, and $1 + \cdot 7 = 1\cdot 7$, therefore

$\frac{1\cdot 7 \times 10 \times 10 \times \cdot 08}{12} = 1\cdot 133$ inches, the space required.

185. Prop. 8. To determine a rule for finding the deflexion of a cast-iron beam, of uniform breadth, when the outline of the depth is a parabola, the strain being equal to 15,300 lbs. per square inch.

The same rules will apply to beams of the form of section shown in *Fig.* 9. and 11. when the breadth is uniform.

186. Case 1. When a beam is supported at the ends, and the load is applied in the middle.

The deflexion for this case is calculated in art. 89. Equation xiv.; and comparing it with

the deflexion of an uniform beam we have $\frac{2}{3} : \frac{4}{3} :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 04\, l^2}{d} =$ the deflexion.

187. Rule. Multiply the square of the whole length of the beam in feet by ·04; divide the product by the middle depth in inches, and the quotient will be the deflexion in inches.

188. Example. Let the depth of a beam be 18·4 inches, and its length 20 feet, which is on the supposition that the beam, of which the depth is found by example to Case 1. Prop. 5. art. 150. is parabolic. By the rule $\frac{20 \times 20 \times \cdot 04}{18 \cdot 4} = \cdot 87$ inches, the deflexion required.

If the beam were of uniform depth, the deflexion would be only half this quantity, or ·435.

189. Case 2. If a parabolic beam, of uniform breadth, be fixed at one end, and the force be applied at the other, the deflexion of the end where the force is applied will be $\frac{\cdot 16\, l^2}{d}\ (1 + r.)$ where l is the length of the part the force acts on in feet, and $r =$ the quotient arising from dividing the length of the fixed part by the length l.

190. Rule. Divide the length, in feet, of the fixed part of the beam, by the length in feet of the part which yields to the force, and add 1 to the quotient. Then multiply the square of the length in feet by the quotient so increased, and also by ·16; divide this product by the middle depth in inches, and the quotient will be the deflexion in inches.

191. Example. Let AB *Fig.* 26. be the beam of a steam engine, the moving force acting at A and the resistance at B, C being the centre of motion; when AC = 12 feet, and CB = 10, and the depth in the middle 30 inches; it is required to determine the space the point A bends through before the full action is exerted on B, the strain being equal to the elastic force of the material.

In this case the length of the part CB, which may be considered as fixed, is 10 feet, and $\frac{10}{12} = \cdot 8\dot{3}$, and $1 + \cdot 8\dot{3} = 1 \cdot 8\dot{3}$; therefore $\frac{12 \times 12 \times 1 \cdot 8\dot{3} \times \cdot 16}{30} = \frac{12 \times 22 \times \cdot 16}{30} = 1 \cdot 408$ inches, the deflexion of the point A.

Few people are aware of the extent of flexure in the parts of engines, and particularly when they are executed in a material which has been considered as nearly inflexible. In a well contrived machine the importance of making the parts capable of transmitting motion and power, with precision and regularity, must be so obvious, that it appears almost incredible how much the laws of resistance have been neglected.

192. Prop. 9. To determine a rule for finding the deflexion of a cast-iron beam of uniform breadth, when the depth at the end is only half the depth at the middle, the strain being equal to 15,300 lbs. on a square inch.

193. Case 1. When a beam is supported at the

ends, and the load is applied in the middle. By art. 93. Equation xviii. $\frac{1 \cdot 09 \iota l^2}{d} =$ DA the deflexion; when this is compared with Equation xii. art. 87. we have $\frac{2}{3} : 1 \cdot 09 :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 0327\, l^2}{d} =$ DA the deflexion.

194. Rule. Multiply the square of the length in feet by ·0327, and the product divided by the depth in the middle in feet, will give the deflexion in inches.

195. Case 2. When a beam is fixed at one end, and the force is applied at the other. In this case $\frac{\cdot 13\, l^2}{d}(1+r) =$ the deflexion.

196. Rule. Calculate the deflexion by the rule, art. 190. except changing the multiplier to ·13 instead of ·16.

197. Prop. 10. To determine a rule for finding the deflexion of a beam, generated by the revolution of a cubic parabola, the strain being equal to 15,300 lbs. on a square inch.

The same rules will apply to any cases where the sections are similar figures, and the cube of the depth every where proportional to the leverage the force acts with.

198. Case 1. When a beam is supported at the ends, and the load is applied in the middle.

By art. 90. Equation xv. $\frac{6 \iota l^2}{5 d} =$ DA the deflexion, and comparing this with Equation xii. we have $\frac{2}{3} : \frac{6}{5} :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 036\, l^2}{d} =$ the deflexion.

199. Rule. Substitute ·036 in the place of ·04 in the rule to Prop. 8. art. 187. and then calculate the deflexion by that rule.

200. Case 2. When a beam is fixed at one end, and the force acts at the other.

In this case $\frac{\cdot 144\, l^2}{d} =$ the deflexion.

201. Rule. In the rule to Prop. 8. art. 190. use ·144 instead of ·16 as a multiplier, and calculate the deflexion by that rule, so altered.

202. Prop. 11. To determine a rule for finding the deflexion of a cast iron beam, of uniform breadth, the depth being bounded by an ellipse; the strain being equal to 15,300 lbs. in a square inch.

If the Equations xii. and xvi. be compared, it will be found that $\frac{2}{3} : \cdot 857 :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 0257\, l^2}{d} =$ the deflexion.

203. Rule. The deflexion may be calculated by the rule to Prop. 8. art. 187. if the multiplier ·0257 be employed instead of ·04.

204. Prop. 12. To determine a rule for the deflexion of a beam of uniform depth, when the breadth is bounded by a triangle, the strain upon a square inch being 15,300 lbs.

From Equation xii. and xiii. art. 87. and 88. we have $\frac{2}{3} : 1 :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 03\, l^2}{d} =$ the deflexion.

205. Case 1. When a beam is supported at the ends, and loaded in the middle.

206. Rule. Calculate by the rule to Prop. 8. art. 187. using ·03 instead of ·04 as a multiplier.

207. Case 2. When a beam is supported at one end, and fixed at the other.

In this case $\frac{\cdot 12 l^2}{d} =$ the deflexion.

208. Rule. Calculate the deflexion by the rule to Prop. 8. art. 190. using ·12 as a multiplier instead of ·16.

208 *(a)*. The rules derived from the twelve preceding propositions are applicable to any kind of material. For example, let it be required to adapt any one of the rules for oak: in the alphabetical table at the end of this Essay, art. OAK, it appears that oak is 0·25 as strong as cast iron; therefore, in a rule for strength, multiply the constant number by 0·25. Thus in the rules to Prop. 1 Case 1. $850 \times 0{\cdot}25 = 212{\cdot}5$ the number to be used in these rules when the material is oak.

Again, oak is twenty-eight times as extensible as cast iron; consequently the deflexion being found for cast iron, twenty-eight times that deflexion will be the deflexion of oak, when it is strained to the extent of its elastic power.

On the Stiffness of Cast Iron.

209. *Definition.* The *stiffness* of a body is its resistance at a given deflexion.

210. Prop. 13. To determine the stiffness of an uniform bar or beam, fixed at one end, to resist a weight at the other.

When a beam is strained to the extent of its elastic force, the weight it will bear, or $W = \frac{212 \cdot 5\, b\, d^2}{l}$ (by art. 116.) and the deflexion under that strain will be $= \frac{\cdot 08\, l^2}{d} \times (1 + r,)$ (by art. 182.) Then, since the deflexion is proportional to the strain, if a be the given deflexion, and w the weight which produces it, we have

$$(1+r)\,\frac{\cdot 08\, l^2}{d} : a :: \frac{212 \cdot 5\, b\, d^2}{l\,(1+r)} : w = \frac{2656\, b\, d^3 a}{l^3\,(1+r)}.$$

$$\text{or}\ \frac{w\, l^3\,(1+r)}{2656\, a} = b\, d^3. \qquad \text{(xxvii.)}$$

Where l = the length in feet, a the deflexion in inches, b and d the breadth and depth in inches, and w the weight in pounds; and r=the length of the fixed part divided by l. When $r=1$, the lengths are equal, and $(1+r)=2$.

211. If the fixed part be of considerable bulk in respect to the other, we may neglect its effect on the deflexion, and in that case $\frac{w\, l^3}{2656\, a} = b\, d^3$. (xxviii.)

Examples of the Application of this Proposition to Beams of Pumping Engines, Cranks, and Wheels.

Beams of Pumping Engines.

212. Example. Let it be required to determine the breadth of a beam for a pumping engine, its whole length being 24 feet, and the parts on each side of the centre of motion equal; and the straining

force 30,000 lbs. the deflexion not to exceed 0·25 inches.

By Equation xxvii. art. 210. $\frac{w\,l^3(1+r)}{2656\,a}=$ $\frac{30,000\quad 12^3\times(1+1)}{2656\times\cdot 25}=b\,d^3=156,145$. If the breadth be made 5 inches, the depth should be 31·5 inches; for $3\cdot 15^3\times 5=156,279$, which very little exceeds 156,145.

If the depth at the middle be double the depth at either end, use 1640 as a divisor instead of 2656.

Cranks.

213. Example. If the force acting upon a crank be 6000 lbs. and its length be 3 feet, to determine its breadth and depth so that the deflexion may not exceed one-tenth of an inch.

By Equation xxviii. art. 211. $\frac{W\,l^3}{2656\,a}=\frac{6000\times 3^3}{2656\times 1}=$ $b\,d^3=610$ nearly.

If the breadth be made 3 inches, the depth should be 6 inches, for the cube of $6\times 3=648$.

When the depth at the end where the force acts is half the depth at the axis, use 1640 instead of 2656 for a divisor.

Wheels.

214. For wheels, if N be the number of arms, or radii, our equation should be $\frac{W\,l^3}{2656\,N\,a}=b\,d^3$.

215. Example. Let the greatest force acting at the circumference of a spur-wheel be 1600 lbs. the

radius of the wheel 6 feet, and the number of arms 8; and let the deflexion not exceed one-tenth of an inch.

Then by the Equation, art. 214. $\frac{W l^3}{2656 N a} = \frac{1600 \times 6^3}{2656 \times 8 \times \cdot 1} = b d^3 = 163.$

If we make the breadth 2·5 inches, then $\frac{163}{2 \cdot 5} = 65 \cdot 2 = d^3$, and the cube root of $65 \cdot 2 = 4 \cdot 03$ inches, nearly, for the depth or dimension of each arm, in the direction of the force.

When the depth at the rim is intended to be half that at the axis, use 1640 as a divisor instead of 2656 for a divisor.

If a wheel be strained till the arms break, the fracture takes place close to the axis; there is a sensible strain at the part of the arm near the rim, but it is so small in respect to that at the axis, that its effect is neglected in our rule.

216. Prop. 14. To determine the stiffness of an uniform bar, or beam, supported at the ends, to resist a cross strain in the middle.

By art. 106. and 174. if the beam be rectangular $\frac{\cdot 02\, l^2}{d} : a :: \frac{850\, b\, d^2}{l} : w = \frac{42500\, b\, d^3\, a}{l^3}$ or $\frac{w\, l^3}{42500\, a} = b\, d^3.$ (xxix.)

217. If $a = \frac{l}{40}$ of an inch it becomes $\cdot 00094\, w\, l^2 = b\, d^3$, which was made $\cdot 001\, w\, l^2 = b\, d^3$ to calculate the table, art. 5.

218. When the beam is a solid cylinder. By art. 129. and 174. we obtain $\frac{\cdot 02\, l^2}{d} : a :: \frac{500\, d^3}{l} : w = \frac{25000\, d^4\, a}{l^3}$, or $\frac{w\, l^3}{25000\, a} = d^4$. (xxx.)

219. In a hollow cylinder or tube, (by art. 141. and 174.) $\frac{\cdot 02\, l^2}{d} : a :: \frac{500\, d^3 (1-N^4)}{l} : w =$

$\frac{25000\, d^4\, a\, (1-N^4)}{l^3}$, or $\frac{W\, l^3}{25000\, a\, (1-N^4)} = d^4$. (xxxi.)

In these equations l is the length between the supports in feet, d the depth or diameter in inches, b the the breadth in inches, a the deflexion in inches, and w the weight producing it in pounds.

220. Example. Required the diameter of a solid cylindrical shaft, 21 feet in length, that would not be deflected more than half an inch by a weight of 31 cwt. or 3472 lbs. applied in the middle.

By Equation xxx. art. 218. $\frac{w\, l^3}{25000\, a} = \frac{3472 \times 21^3}{25000 \times \cdot 5} = d^4 = 2572$. or $d = 7\cdot 12$ inches, the diameter required.

221. Example. Required the diameter of a hollow shaft, 21 feet in length, the interior diameter seven-tenths of the exterior one, that would not be deflected more than half an inch by a load of 3472 lbs. applied in the middle of the length.

By Equation xxxi. art. 219. $\frac{w\, l^3}{25000\, a (1-N^4)} =$

$\frac{3472 \times 21^3}{25000 \times \cdot 5 \times (1 - \cdot 7^4)} = d^4 = 3384$. therefore $d = 7\cdot 627$ inches, and the interior diameter $= 7\cdot 627 \times \cdot 7 = 5\cdot 34$. inches.

Resistance to Torsion.

222. *Definition.* The resistance which a shaft or axis offers to a force applied to twist it round is called the resistance to *Torsion.*

223. If a rectangular plate be supported at the corners A and B, *Fig.* 29. *Plate* IV. and a weight be suspended from each of the other corners C, D; then, the strains produced by loading it in this manner will be similar to the twisting strain which occurs in shafts, and the like. In a cast-iron plate the fractures would take place in the directions AB and CD at the same time; but, before the fracture, the one of the strains will serve as a fulcrum for the other; and the resistance to the forces at C and D will be sensibly the same as if the plate were supported upon a continued fulcrum in the direction AB.

Hence the strain may be considered a cross strain of the same nature as has been investigated in art. 77. and aD or cC the leverage the force at D or C acts with, the breadth of the strained section being AB.

To find the breadth of the section of fracture, and the leverage in terms of the length and breadth of the plate, we have AB, the breadth, and by similar triangles $\frac{AD \times BD}{AB} = Da$ the leverage. These values of the leverage and breadth being substituted in the equation art. 79. it becomes $W \times \frac{f b d^2}{6 l} = \frac{f d^2 \times AB \times AB}{6 \times AD \times BD}$; or because, $AB^2 = BD^2 + AD^2$ $W = \frac{f d^2}{6} \frac{BD^2 + AD^2}{AD \; BD}$.

224. But when a force acts upon a shaft it is usually at the circumference of a wheel upon that shaft, and if R be the radius of the wheel, then $\frac{2RW}{BD}$ = the force collected at the surface of the shaft; therefore, substituting this in the place of W, in the equation above, we have $\frac{2RW}{BD} = \frac{fd^4}{6} \times \frac{BD^2 + AD^2}{AD \times BD}$ or $W = \frac{fd^4}{12R} \times \frac{BD^2 + AD^2}{AD}$.

225. When a shaft is square, and its length l in feet, its side d in inches, and the leverage R in feet, then, from equation, art. 80. we obtain $W = \frac{fd^2}{3458}Rl \times (^2d^2 + 144l^2)$. And when $f = 15{,}300$ lbs. $W = \frac{8{\cdot}85\,d^2}{Rl} \times (d^2 + 72\,l^2)$.

226. In a cylindrical shaft the section of fracture is an ellipse, and when l and R are in feet, and $f = 15{,}300$, d being the diameter of the shaft in inches, we have by art. 82. $W = \frac{5{\cdot}2\,d^2}{Rl} \times (d^2 + 144\,l^2)$.

227. It may be shown, by the principles of maxima and minima, that there is a particular line of fracture where the resistance to torsion is a minimum; in a cylindrical body this happens when $12\,l = d$; that is, when the length is equal to the diameter.

Consequently, in all cases where the length exceeds the diameter, the equation in art. 226. should be applied in the form $\frac{125{\cdot}8\,d^3}{R} = W$.

As the equation reduces to this form by substituting $\frac{d}{12}$ for l.

The reader may find further information on this part of the subject in Dr. Young's Lectures on Nat. Philos. vol. i. p. 140, 141. Dr. Brewster's Edinburgh Encyclopedia, art. Mechanics, p. 544 to 549.

Practical Rule and Example for the Resistance to Twisting.

228. When an axis or shaft is cylindrical, by solving the equation, art. 226. and reducing it to a convenient form for calculation, we obtain $d=l^2\left(\sqrt{\frac{RW}{5{\cdot}2\,l^3}+5184}-72\right)$; which expressed in words is the rule.

Rule. Multiply the twisting force in pounds by the radius in feet of the wheel to the circumference of which it is applied; and divide this product by 5·2 times the cube of the length in feet; to the quotient thus obtained add the number 5,184; and extract the square root of the sum: from the square root subtract 72, and multiply the remainder by the square of the length in feet, the product will be the diameter in inches.

229. Example. Let it be required to find the diameter of a shaft for the water wheel, the length of the shaft from the water wheel to the cog wheel being 6 feet, the radius of the water wheel 9 feet, and the greatest force that it will be exposed to at the circumference 2000 lbs.

In this case $\frac{9 \times 2000}{5 \cdot 2 \times 6^3} = \frac{18000}{1123 \cdot 2} = 16$ nearly, and $16 + 5184 = 5200$, of which the square root is $72 \cdot 12$; and subtracting 72 we have a remainder of $\cdot 12$, which multiplied by 36, the square of the length, gives $4 \cdot 32$ inches, the diameter required.

Calculating by the equation in art. 227. the diameter should be $5\frac{1}{4}$ inches. Hence it appears that such a shaft would always be sufficient to resist the twisting force, when made strong enough to resist the lateral stress.

Of the Strength of Cast Iron Columns, Pillars, or other Supports compressed, or extended, in the Direction of their Length.

230. If the length of a column be considerable with respect to its diameter, under a certain force it will bend; but when it becomes too short to bend, its strength is only limited by the force which would crush it. Considering however that it is imprudent to load, even a short column, beyond its elastic force, an inquiry respecting the phenomena of crushing would lead to nothing useful.

Let AA' be a column, *Fig.* 30. supported at A', and supporting a load at A; and let this load have produced its full effect in straining the column. Let E be the neutral axis, B and D the centres of resistance, and AF the direction of the straining force. Draw dD parallel to AF, then, by the principles of statics, we have $d\mathrm{D} : \mathrm{DA} :: \mathrm{W}$ (the weight) : $\frac{\mathrm{W.DA}}{d\mathrm{D}}$ = the com-

pressive force in the direction AD. Also, DA : AF : : $\frac{W.DA}{dD} : \frac{W.AF}{dD}$ = the vertical pressure at D.

But, by similar triangles, BD : BF : : dD : AF= $\frac{BF.\,dD}{BD}$, therefore $\frac{W.AF}{dD}=\frac{W.BF}{BD}$. (xxxii.)

231. In a similar manner it may be shown, that the strain at B is expressed by $\frac{W.\overline{BF-BD}}{BD}$. wherein it is obvious than when BD=BF this strain is nothing, that is, when the direction of the straining force passes through the point D, or the neutral axis coincides with the surface of the block. It also may be observed, that when BF exceeds BD this strain is expressed by a positive quantity indicating extension, but when BF is less than BD it is negative, indicating that it is a resistance to compression. If BF=$\frac{1}{2}$BD then both points are equally compressed. And from these equations it appears that the neutral axis will never coincide with the axis of the column.

232. It may be shown that the resistance of the section, on either side of the neutral axis, is equal to the force of a square inch multiplied by the area of that section multiplied by the distance of the centre of gravity from the neutral axis, and divided by the distance of the compressed surface from the neutral axis, when B or D is the centre of percussion of the section*.

233. Let x be the distance of the neutral axis from

* Emerson's Mechanics 4to. edit. Prop. 77.

the middle of the depth; y=EF the distance of the direction AF of the straining force from the middle of the depth; d=the depth, b=the breadth, and f the resistance of a square inch; then, the area of the compressed part of the section will be $\overline{\tfrac{1}{2}d+x}\times b$, and the extended part of the section $(\tfrac{1}{2}d-x)\,b$. Therefore, if $n\,(\tfrac{1}{2}d+x)$ and $n\,(\tfrac{1}{2}d-x)$ be the distances of the centres of percussion from the neutral axis, and $m\,(\tfrac{1}{2}d+x)$ and $m(\tfrac{1}{2}d-x)$ the distances of the centres of gravity, we shall have $\frac{W.BF}{BD}\times\frac{mfb(\frac{1}{2}d-x)^2}{(\frac{1}{2}d+x)}=$ $\frac{W.(BF-BD)}{BD}\times\frac{mfb(\frac{1}{2}d+x)^2}{(\frac{1}{2}d+x)}$, or $BF\times(\tfrac{1}{2}d-x)^2=$ $(BF-BD)\times(\tfrac{1}{2}d+x)^2$; and when the proper substitutions are made, this equation reduces to $x^2\,(2-3n)+2\,y\,x-\tfrac{1}{4}n\,d^2=0$.

234. In a rectangular section $n=\frac{2}{3}$, and consequently we find by the preceding equation $x=\frac{d^2}{12y}$. Also, since in this case $m=\tfrac{1}{2}$, we have an equilibrium between the compressing force, and the resistance to compression, when $\frac{W.BF}{BD}=\frac{fb}{2}(\tfrac{1}{2}d+x)$; and substituting for BF, BD, and x their proper values, this equation becomes $W=\frac{f\,b\,d^2}{d+6y}$. (xxxiii.)

This equation applies to short columns, or blocks, of which the length is not more than ten or twelve times the least dimension of the section; and from it is derived the following practical rule.

To find the Area of a short rectangular Column or Block to resist a given Pressure.

235. Rule. When the force is to be applied exactly in the axis or centre of the section of the block, divide the pressure or the weight in pounds, by 15,000, and the quotient will be the area of the section of the block in inches. But since this requires a degree of precision in adjusting the direction of the force which it is altogether impossible to arrive at in practice, and when a force presses a block of which $a\,a'$ is the axis, *Fig.* 31. *Plate* IV. it is always probable that the direction AA′ of the force may act upon one edge only of the end of the block, and therefore be at a distance of half the least thickness from the axis; which will reduce the resistance of the block to one-fourth, and consequently the area should always be made four times as great as is determined by this rule.

When the distance of the direction of the force from the axis is determined by the nature of the construction, the following is a general rule.

236. Rule. To the thickness (or least dimension of the section) in inches, add six times the distance of the direction of the force from the axis in inches, and let this sum be multiplied by the weight or pressure in pounds; divide the quotient by 15,300 times the square of the least thickness in inches, and the quotient will be the breadth of the block in inches.

This rule is the Equation xxxiii. art. 234. in words at length, and it applies to resistance to tension as well as to resistance to compression.

237. The writer of the art. Bridge, in the Supplement to the Encyclo. Brit. has shown that when the force acts in the direction of the diagonal of the block, as is shown in *Fig.* 32. the strain will be twice as great as when the same force acts in the direction of the axis*. Now the reader will be satisfied, that, in consequence of settlements, or other causes, a column is always liable to be strained in this manner; and therefore will carefully avoid enlarging the ends of his columns, under the notion of gaining stability, for the effect of the straining force will be still more increased by such enlargement in the event of a change of direction from settlement; as in *Fig.* 33. In my Treatise on Carpentry, I have recommended circular abutting joints to lessen the effect of a partial change in the position of the strained pieces†, an idea which appears to have occurred, in the first instance, to Serlio‡.

238. A general solution of the equation expressing the stress and strain, when the column is cylindrical, is complicated, but in one particular case the result is extremely simple; that is, when the neutral axis is in one of the surfaces of the column. If d be the diameter of the column, then $\cdot 7854 d^2 =$ the area, and $\frac{1}{2}d =$ the distance of the centre of gravity, and therefore $\frac{\text{WBF.}}{\text{BD}} = \frac{\cdot 7854\, d^2 f}{2}$. But when the neu-

* Supp. to Ency. Brit. art. Bridge, Prop. I. p. 499.

† Tredgold's Elementary Prin. Carpentry, Sect. IX. p. 142.

‡ Serlio's Architecture, Lib. I. p. 13. Paris, 1545.

tral axis is in the surface of the cylinder $BF = BD$, or $W = \frac{\cdot 7854\, d^2 f}{2}$. In this case the distance of the direction of the force from the axis of the column will be $\frac{1}{4}$ of the diameter, the centre of percussion being $\frac{1}{4}\, d$ distant from the neutral axis.

239. Hence it appears, that when the distance of the direction of the force from the axis is $\frac{1}{4}\, d$, the strength of a cylinder is to that of its circumscribed square prism, as seven times the area of the cylinder, to eight times the area of the prism; or nearly as $5 \cdot 5 : 8$; or as $1 : 1 \cdot 46$ nearly.

When the neutral axes are at or near the axes of the pieces, the ratio of the strength of the cylinder to that of the prism becomes $\frac{3 \times \cdot 7854}{4} : 1$, or as $1 : 1 \cdot 7$, as has been shown by Dr. T. Young*; consequently in a column, when both the resistance to compression and extension are brought into action, the ratio varies between $1 : 1 \cdot 46$ and $1 : 1 \cdot 7$; the mean being nearly $1 : 1 \cdot 6$.

240. If a support be compressed in the direction of its length, and the deflexion be sufficient to sensibly increase the distance of the direction of the force from the axis, in the middle of the length of the support, it is evident that the strain will be increased; and since the curvature in practical cases will be very small, we may suppose it to be an arc of a circle. In a circle the square of the length of the chord, in a small segment, is sensibly equal to

* Dr. Young's Lectures on Nat. Philos. Vol. ii. art. 339, B.

the radius × 8 times the versed sine; or, $\frac{l^2}{8\delta}$ = radius. The deflexion will be greatest when the neutral axis coincides with the axis, and taking this extreme case, we shall have this analogy; as the alteration of the length of the concave side, is to the original length, so is the $\frac{1}{2}$ depth to the radius of curvature; or, $\epsilon : 1 :: \frac{d}{2}$: radius $= \frac{d}{\epsilon}$. Therefore $\frac{l^2}{8\delta} = \frac{d}{2\epsilon}$; and $\delta = \frac{l^2\epsilon}{4d}$ = the deflexion in the middle.

241. Let the distance of the direction of the force from the axis, when first applied, be denoted by a; then y in the preceding articles (art. 233, 234) will be equal to $a + \frac{l^2\epsilon}{4d}$; consequently $W = \frac{fbd^2}{d+6y} = \frac{fbd^2}{d+6a+\frac{6l^2\epsilon}{4d}}$.

In cast iron $f = 15{,}300$lbs. and $\epsilon = \frac{1}{1204}$, (art. 106. and 174.) therefore if l be the length in feet, b, d, and a in inches, we obtain the following practical formula, for the strength of a rectangular prism, viz.

$$W = \frac{15{,}300bd^2}{d+6a+\frac{\cdot 18l^2}{d}}. \qquad \text{(xxxiv.)}$$

242. If $a = o$, or the direction of the force coincides with the axis, then the rule becomes

$$W = \frac{15{,}300bd^2}{d+\frac{\cdot 18l^2}{d}} \qquad \text{(xxxv.)}$$

243. As an approximate rule for the strength of a cylinder to resist compression in the direction of its length, we have $W = \dfrac{15{,}300\, d^3}{1{\cdot}6\left(\dfrac{d + {\cdot}18 l^2}{d}\right)} = \dfrac{9562{\cdot}5 d^4}{d^2 + {\cdot}18 l^2}$. (xxxvi.)

244. And if the direction of the force be a inches distant from the axis, the rule is $W = \dfrac{9562{\cdot}5 d^3}{d + 6a + \dfrac{{\cdot}18 l^2}{d}}$. (xxxvii.)

245. Example. Required the weight that could be supported, with safety, by a cylindrical column, the length being 11 feet, and the diameter 5 inches, and supposing it probable that the force may act in the direction AA′ *Fig.* 31. at the distance of half the diameter from the axis?

In this example Equation xxxvii. art. 244. should be used; and therefore $\dfrac{9562{\cdot}5\, d^3}{d + 6\, a + \dfrac{{\cdot}18\, l^2}{d}} = \dfrac{9562{\cdot}5 \times 5^3}{5 + 15 + \dfrac{{\cdot}18 \times 11^2}{5}}$ $= W = 49{,}480$ lbs. or a little above 22 tons.

In this manner may be calculated the strength of story posts for supporting buildings. When they are for houses ample allowance should be made for the weight of crowded rooms, and also for the weight of goods in warehouses.

246. Example. Required the weight that may be suspended by a bar of cast iron of 4 inches by 8 inches; under the supposition that the direction of the strain will be in one of the wide surfaces of the bar? Equation xxxiii. art. 234. applies to this case,

wherein y is equal 2 inches, or half the least dimension of the bar, that being the distance the direction of the force is supposed to be from the axis; and therefore $\frac{f\,b\,d^2}{d+6y} = \frac{15,300 \times 8 \times 4^2}{4+12} = 122,400$ lbs. the weight required. See the rule in words at art. 236.

When it is considered that a very small degree of inaccuracy in fitting the connexion may throw the strain all on one side of the bar, the prudence of following this mode of calculation will be apparent.

247. Example. It is proposed to determine the compression a curved rib will sustain in the direction of its chord; the greatest distance of the axis of the rib from the chord line being 6 inches, the size of the rib 3 inches square; and the length of the chord line 5 feet.

By Equation xxxiv. art. 241. $W = \frac{15,300\, b\, d^2}{d + 6a + \frac{\cdot 18\, l^2}{d}} =$

$$\frac{15,300 \times 3^4}{3 + (6 \times 6) + \frac{\cdot 18 \times 5^2}{3}} = 30,600 \text{ lbs.}$$

Since the fifth Section was printed, Mr. Telford has given me the results of the experiments made by Mr. William Reynolds, of Ketley, in Shropshire; whence it appears that the estimate I had quoted in art. 63. of the power of cast iron to resist compression, is erroneous. The experiments were made for Mr. Telford, and

	per square inch.
A cube of 1-4th of an inch of soft gray metal was crushed by 80 cwt.	= 143,360 lbs.
Ditto of gun-metal was crushed by 200 cwt ..	= 350,400 lbs.

SECTION VII.

OF THE STRENGTH OF CAST IRON TO RESIST AN IMPULSIVE FORCE.

248. The moving force of a body, or of a part of a machine, ought to be balanced by the elastic force of the parts which propagate the motion; for if the effect of the moving force be greater than the elastic force of the parts, some of them will ultimately break; besides, a part of the power of the machine will be lost at each stroke.

And since increasing the mass of matter to be moved, increases the friction in a machine, it is an advantage to employ no more material in its moving parts, than is absolutely necessary for strength; but, in other parts exposed to straining forces, it is desirable that the materials should always be capable of resisting the strains, with as small a degree of flexure as is convenient, because steadiness is, in the fixed parts of machines, a most desirable property.

A beam resists a moving force, as a spring, by yielding and opposing the force as it yields, till it

finally overbalances it*; and hence it is that a brittle, or very stiff body breaks, because it does not yield sufficiently for destroying the force.

As the resistance of a beam under different degrees of flexure can be calculated, the effect of that resistance in the destruction of motion may be estimated by the principles of dynamics; such inquiries are usually managed by the method of fluxions; but, not being satisfied with the manner of establishing the principles of that method, though I have no doubt of the correctness of results obtained by it, I shall briefly deduce the rules of this section by another mode of calculation.

249. If the intensity of a force be variable, so that the action upon the body moved at any point be directly as some power, n, of the distance from a point B, *Fig.* 23. towards which it moves. Then, if the intensity of the force at A be equal P, the intensity at any point C will be $\frac{\overline{CB}^n.P}{\overline{AB}^n}$. For, by the definition,

$$\overline{AB}^n : \overline{CB}^n :: P : \frac{\overline{CB}^n.P}{\overline{AB}^n}.$$

Put S to denote the space AB; and conceive this space S to be divided into m equal parts, denoting any one of these parts by x; and, in consequence of the smallness of these parts, if we take the mean between the intensity at the beginning and that at the end of each part, and consider each of these means an uniform intensity for the space it was calculated for,

* For a machine to produce the greatest effect, the time of bending the beam should be as small as possible.

then these uniform intensities may be represented by the following progression.

$$\frac{P}{2\,S^n}\times\left(\overline{0+x^n}+\overline{x^n+2^n x^n}+\overline{2^n x^n+3^n x^n}+\cdots\overline{\overline{m-1}^n x^n+m^n x^n}\right)$$

$$\text{Or, } \frac{Px^n}{2S^n}\left(\overline{0+1^n}+\overline{1^n+2^n}+\overline{2^n+3^n}+\cdots\cdots\overline{\overline{m-1}^n+m^n}\right).$$

250. It is shown by writers on dynamics, that when the intensity of a force is uniform, the square of the quantity of force accumulated or destroyed is directly as the intensity multiplied by the quantity of matter moved, and by the space moved through*. Therefore, making W = the quantity of matter, and g a constant quantity to reduce the proportion to an equation, we find the square of the forces accumulated or destroyed, in the space S, may be exhibited by the progression $\frac{gPWx^{n+1}}{2\,S^n}\left(\overline{0+1^n}+\overline{1^n+2^n}+\cdots\cdots\cdots\right.$ $\left.\overline{\overline{m-1}^n+m^n}\right)$.

And, from the principles of the method of progressions†, the accurate value of the square of the force accumulated or destroyed in the space S is $\frac{gPWS}{n+1}$.

251. When $n=o$ or the intensity is uniform, the square of the accumulated force is gPWS.

252. The force of gravity near the earth's surface is nearly uniform, and in this case we know from

* Dr. C. Hutton's Course of Math. Vol. ii. p. 136. 5th Ed.

† See Philosophical Magazine, Vol. lvii. p. 201.

experiments on falling bodies that $g = 64\frac{1}{3}$, and $P = W$ the weight of the body; therefore, $64\frac{1}{3}W^2S$ = the square of the accumulated force, and $64\frac{1}{3}$ may be substituted for g.

253. If $n = 1$, we have $\frac{gPWS}{n+1} = \frac{64\frac{1}{3}PWS}{2} = 32\frac{1}{6}PWS$,

and as, in the resistance of beams, the intensity at any deflexion is directly as the deflexion, the quantity $32\frac{1}{6}PWS$ represents the square of the force destroyed in producing a deflexion equal to S. That is, when a beam is supported at both ends, and S = the deflexion in the middle, in decimal parts of a foot, then $32\frac{1}{6}PWS$ = the force that would be destroyed in producing the flexure S; where P is the weight that would produce the deflexion S.

Having considered the effect of the resisting force of the material, in destroying an impulsive force, we must now consider the circumstances which take place in the different cases occurring in practice.

254. If the blow be made by a falling body in the direction of gravity, and the weight of the falling body be w, and its velocity at the time of impact be v, then by the laws of collision, in the case of equilibrium, $vw = \sqrt{32\frac{1}{6}PS(W+w)}$. (i).

In which equation the small acceleration that would be produced by the action of gravity on the mass $W + w$, during the flexure of the beam, is neglected.

255. If the blow were made horizontally by a body of the weight w, moving with a velocity v, then the

equation is correct; and even in the first case it is accurate enough for practical purposes.

256. If the blow were made bya weight w, falling from a given height h, we have by the laws of gravity, $v = \sqrt{64\frac{1}{3}h}$; therefore, $w\sqrt{64\frac{1}{3}h} = \sqrt{32\frac{1}{6}PS(W+w)}$ or, $2w^2h = PS\,(W+w.)$ (ii.)

257. When the strain is occasioned by a force of an intensity F, and velocity v, such for example as would be occasioned by the sudden derangement of a machine in motion with the velocity v, and force F, then $Fv = \sqrt{32\frac{1}{6}PSW}$; or, $F^2v^2 = 32\frac{1}{6}PSW$. (iii.)

The last equation is applicable to the beams of steam engines, and in general to reciprocating movements in machines, such as the connecting-rods, cranks, &c.

If a body be previously in motion in the direction of the impulsive force, the force Fv should be the difference between the forces of the impelling and impelled bodies.

258. These equations flow from the principle that while the elasticity is perfect, the deflexion or extension is as the force producing it, but it also varies according to the manner in which the material is strained. In some cases, of frequent occurrence, the application is shown in the examples.

It will be useful, before we proceed any farther, to inquire what velocity cast iron will bear, without permanent alteration, in order that we may be aware whether such velocity will ever take place in the parts of machines; for if any part of a machine be connected with others that will yield to the force,

and the material be capable of transmitting the motion with greater velocity than the machine moves with, it need be formed only for resistance to power or pressure.

259. It has been shown that $\sqrt{32\frac{1}{4}\mathrm{PWS}}$ is equal to the greatest force an elastic body can generate or destroy; (art. 253.) if it were exposed to a greater force, its arrangement would be permanently altered. Now, if V be the greatest velocity the body is capable of transmitting, we have $\sqrt{32\frac{1}{4}\mathrm{PWS}} = \mathrm{VW}$,

or, $\sqrt{\frac{32\frac{1}{4}\mathrm{PS}}{\mathrm{W}}} = \mathrm{V}$. (iv.)

260. It has also been shown, that in cast iron, the cohesive force $f = 15{,}300$ lbs. (art. 106.) and the extension $\iota = \frac{1}{1204}$; (art. 174.) and since $\mathrm{S} = l\iota$, (by art. 74.) and $\mathrm{P} = bdf$, (art. 73.) and $lbdp = \mathrm{W}$, where $p = 3{\cdot}2$ lbs. the weight of a bar of iron twelve inches long and one inch square, when a bar is strained in the direction of its length. $\frac{\sqrt{32\ \mathrm{PS}}}{\mathrm{W}} = \sqrt{\frac{32\frac{1}{4} \times bdf \times l\iota}{lbdp}}$

$= \sqrt{\frac{32\frac{1}{4} \times 15{,}300}{3{\cdot}2 \times 1204}} = \mathrm{V} = 11{\cdot}3$ feet, per second.

261. If an uniform bar be supported at the ends, we have $\mathrm{P} = \frac{850bd^2}{l}$ (art. 106.) and $\mathrm{S} = \frac{{\cdot}02l^2}{d}$ (art. 174.) also, $\mathrm{W} = \frac{lbdp}{2}$. Consequently, $\sqrt{\frac{32\frac{1}{4}\mathrm{PS}}{\mathrm{W}}} =$ $\sqrt{\frac{32\frac{1}{4} \times 850 \times {\cdot}02 \times 2}{3{\cdot}2}} = \mathrm{V} = 18{\cdot}5$ feet per second, nearly.

262. Hence it is clear that cast iron is capable of sustaining only a very small degree of velocity; and a correct knowledge of this limit is certainly of the first importance in the application of this material in machinery. When cast iron is exposed to an impulsive force in the direction of its length, the velocity of the force must never exceed eleven feet per second; when the force strikes in a direction perpendicular to the length, its velocity should never exceed eighteen feet per second.

263. To illustrate the use of our investigation, or rather, to prevent any one from disappointment, in applying our rules for the resistance to impulsion, it may be useful to consider how they should be applied to the parts of machines. In a machine, the motion is communicated from the impelled to the working point by a certain number of parts, and among these parts one, at least, should be capable of resisting the whole energy of the moving power. If there be many parts to transmit the power, then two or more of them should be capable of resisting the energy of the moving power, and they should be distributed so as to divide the line of communication into nearly equal parts. If the intermediate parts be made sufficient to resist the dead power of the machine, that is, the power without velocity, they will always be strong enough to convey the velocity, if it be less than is stated in the preceding article, to other parts, that will either forward it to the working point, or resist it entirely during a momentary derangement of the action of the machine. To

make all the parts strong enough for this purpose, would often cause a machine to be clumsy, and unfit for any practical use.

264. Let the constant numbers for the strength and deflexion in feet, be f and δ. Then $P = \frac{f b d^2}{l}$, and $S = \frac{\delta l^2}{d}$. Also, let the weight of the beam itself be n times the weight of the falling body. These values being substituted in Equation i. art. 254. we have $vw = \sqrt{32\frac{1}{6}PSW + w} = \sqrt{32\frac{1}{6} l b d f \delta w (n+1)}$.

$$\text{or,} \quad \frac{v^2 w}{32\frac{1}{6} l f \delta \, (n+1)} = bd. \qquad \text{(v.)}$$

265. If the like substitutions be made in Equation iii. art. 257. we obtain $F^2V^2 = (32\frac{1}{6}PSW) = 32\frac{1}{6} l b d f \delta W$, and if $lbdp$ be the weight of the mass of the beam the force acts upon, then $\frac{FV}{\sqrt{32\frac{1}{6} f \delta p l}} = bd$. (vi.)

Practical Rules and Examples.

266. Prop. 1. To determine a rule for finding the dimensions of a beam to resist the force of a body in motion.

It is evident by Equation v. art. 264. that the error which would arise from neglecting to allow for the effect of the weight of the beam itself, would always be on the safe side in calculating the dimensions of a beam to resist an impulsive force; and since, by such neglect, the rule is reduced to a very simple form, instead of a very complicated one, I shall apply the equation under the form $\frac{v^2 w}{32\frac{1}{6} l f \delta} = bd$.

267. Case 1. When the beam is uniform and supported at the ends. In this case $f = 850$. (see art. 106.) and δ in feet $= \frac{\cdot 02}{12}$ (by art. 174.) hence, $32\frac{1}{6}f\delta = 45\cdot 5$; or, $\frac{v^2 w}{45\cdot 5 l} = bd$.

268. Rule. Multiply the weight of the falling body in pounds by the square of its velocity in feet, per second ; divide this product by 45·5 times the length in feet, and the quotient will be the area in inches.

The depth should be at least sufficient to render the beam capable of supporting its own weight, added to the weight of the falling body, which may be readily found by Table II. art. 7.

269. If the height of the fall be given instead of the velocity of the falling body, then instead of multiplying by the square of the velocity, multiply by sixty-four times the height of the fall.

270. Example 1. To determine the area of a cast-iron beam that would sustain, without injury, the shock of a weight of 170 lbs. falling upon its middle with a velocity of eight feet per second, the distance between the supports being twenty-six feet. By the rule $\frac{170 \times 8^2}{45\cdot 5 \times 26} = 9\cdot 6$ inches the area required.

Hence, if we make the depth six inches, the breadth will be 1·6 inches, and the beam would sustain a pressure of 1,881 lbs. (see Table II.) to produce the same effect as the fall of 170 lbs. It may also be observed, that half the weight of the

beam is 400 lbs. making 570 lbs. for the pressure the beam would have to sustain after the velocity was destroyed, which is not quite one-third of the weight the beam would bear.

271. Example 2. If a bridge of thirty feet span were formed on beams of cast iron, of what area should the section of these beams be, so that any one of them might be sufficient to resist the impulsive force of a waggon-wheel falling over a stone three inches high, the load upon that wheel being 3360 lbs.?

The height of the fall being ·25 feet, the square of the velocity acquired by the fall will be $64 \times \cdot 25 = 16$; therefore, $\frac{3360 \times 16}{45 \cdot 5 \times 30} = 39 \cdot 38$ inches, the area required.

This area is nearly 40 inches; suppose it 40, then $40 \times 15 \times 3 \cdot 2 = 1920$ lbs. = half the weight of the beam, (that is, the area, in inches, multiplied by half the length in feet, multiplied by 3·2 lbs. the weight of a piece of cast iron, one foot in length, and an inch square,) consequently, $1920 + 3360 = 5280$ lbs. the whole effective pressure on the beam, after the velocity is destroyed. If we were to make the beam twenty inches deep, and two inches in thickness, it may be found by Table II. that the deflexion would be ·9 of an inch, and it would require a pressure of 45,328 lbs. to produce the same effect as the fall of the wheel, above eight times the pressure of the load and weight.

272. Case 2. When a beam is supported at the

ends, the breadth uniform, and the outline of the depths an ellipse.

This case applies to bridges or beams to withstand an impulsive force at any point of the length. By art. 107. $f = 850$, and by art. 102. δ in feet $= \frac{\cdot 0257}{12}$; therefore the equation $\frac{v^2 w}{32\frac{1}{6} f \delta l} = b\,d$, in art. 266. becomes $\frac{v^2 w}{58\cdot 5\,l} = b\,d$.

273. Rule. Calculate by the rule, art. 268. with $58\cdot 5$ as a divisor instead of $45\cdot 5$.

274. Case 3. When the breadth and depth of a beam are uniform, and the section is as *Fig.* 9. *Plate* I. and the beam supported at the ends.

In this case $f = 850\,(1 - q\,p^3)$ by art. 148. and $\delta = \frac{\cdot 02}{12}$ feet by art. 174.; hence the equation (art. 266.) $\frac{v^2 w}{32\frac{1}{6} f \delta l} = \frac{v^2 w}{45\cdot 5\,(1 - p^3 q)\,l} = b\,d$, consequently the power of a beam to resist impulsive force, when the quantity of material is the same, is considerably increased by giving this form to the section.

275. Case 4. If a beam of the form of section shown in *Fig.* 9. be the elliptical form of equal strength, (see *Fig.* 24. *Plate* III.) then

$$\frac{v^2 w}{58\cdot 5\,l\,(1 - p^3 q)} = b\,d,$$

when the beam is supported at both ends, and the impulsive force acts at any point of the length.

276. Case 5. In an open beam, as *Fig.* 11. *Plate* II.

we may consider the beam as bounded by a semi-ellipse, when the breadth is uniform, and in this case

$$\frac{v^2 w}{58\cdot5\, l\,(1-p^3)} = b\, d.$$

277. Example. To determine the area of the section of an open girder, that would sustain the shock of 300 lbs. falling from a height of one foot, the length between the support being 26 feet, and the depth of the open part, ·7 times the whole depth.

In this example $\frac{v^2 w}{58\cdot5\, l\,(1-p^3)} = \frac{64 \times 300}{58\cdot5 \times 26(1-\cdot343)} =$ 20 inches nearly. This is perhaps as great an impulsive force as it is probable a girder for a room will be likely to be exposed to; and since this area of section would not be sufficient for the greatest pressure, it appears unnecessary to calculate the effect of moving force in the construction of girders.

278. Prop. 2. To determine a rule for finding the dimensions of an uniform beam to resist a moving force.

This proposition applies to the parts *of machines*; and as there are few people engaged in the construction of powerful machines that are not competent to apply an equation, I shall in this part give the rules in the form of equations only.

279. Case 1. When an uniform beam is supported at the ends, and the moving force acts at the middle of the length.

By art. 106. $f = 850$, and by art. 174, $\delta = \frac{\cdot02}{12}$ feet, and since 3·2 lbs. = the weight of one foot in

length, one inch square, we shall have $p = \frac{3 \cdot 2}{\cdot 2}$; therefore $\frac{\mathrm{FV}}{l\sqrt{32\frac{1}{6}}\, f\, \delta\, p}$ (art. 265.) $= \frac{\mathrm{FV}}{l\sqrt{32\frac{1}{6} \times 850 \times \frac{\cdot 02}{12} \times \frac{32}{2}}} =$

$$\frac{\mathrm{FV}}{8 \cdot 6\, l} = b\, d.$$

280. Rule. When F is the force in pounds, V its velocity in feet per second, *l* the whole length in feet between the supports, and *b* the breadth, and *d* the depth in inches, then

$$\frac{\mathrm{FV}}{8.6\, l} = b\, d.$$

281. Case 2. When an uniform beam rests upon a centre of motion, and the moving force acts at one end, and is opposed by a greater resistance at the other end.

By art. 116. $f = 212$, and by art. 182. $\delta = \cdot 08\ (1 + r,)$ and $p = \frac{3 \cdot 2}{2}$, hence $\frac{\mathrm{FV}}{l\sqrt{32\frac{1}{6}\, f\, \delta\, p}} = \frac{\mathrm{FV}}{8 \cdot 6\, l \sqrt{1 + r}} = bd.$

282. Rule. Make F = the force in pounds, V its velocity in feet per second, $l =$ the length in feet between the centre of motion and the point where the force acts, and $l' =$ the length in feet between the centre of motion and the point of resistance; *b* and *d* being the breadth and depth in inches; then

$$\frac{\mathrm{FV}}{8 \cdot 6\, l \sqrt{1 + \frac{l'}{l}}} = b\, d.$$

283. If $l = l'$ we have $\frac{\mathrm{FV}}{8 \cdot 6\, l \sqrt{2}} = \frac{\mathrm{FV}}{12 \cdot 2\, l} = b\, d.$

284. Example. To determine the area of the section of a beam for a steam engine, when it is to be of uniform depth; the length 24 feet, the centre of motion in the middle of the length; the pressure upon the piston 5000 lbs. and its greatest velocity four feet per second.

By art. 283. $\frac{FV}{12 \cdot 2\, l} = bd = \frac{5000 \times 4}{12 \cdot 2 \times 12} = 137$ inches nearly.

If this beam were made 30 inches deep, the deflexion by such a strain would be about 8-10ths of an inch, and the breadth would be $\frac{137}{30} = 4 \cdot 57$ inches, and such a beam would bear a weight of about 12 times the pressure on the piston, without destroying its elastic force.

285. Prop. 3. To determine a rule for finding the area of the middle section of a parabolic beam to resist a moving force when the breadth is uniform.

The motion communicated to the arm of a lever is the same as if its whole weight were collected at its centre of gravity; and as the length of the arm is to the distance of its centre of gravity, so is the mass to the effect of that mass collected at the extremity. Therefore, when the distance of the centre of gravity is some part of the length, the effect of the mass of the arm will be the same part of the whole of its weight when acting at the extremity.

286. Case 1. When a parabolic beam is supported at both ends, and the moving force acts at the middle of the length.

By art. 106. $f = 850$, and by art. 186. $\delta = \frac{\cdot 04}{12}$ feet. Also, because the area of a parabolic beam is $\frac{2}{3}$ of one uniformly deep*, and distance of the centre of gravity from the centre of motion is $\frac{3}{5}$ of the length†; we have $p = 3\cdot 2 \times \frac{2}{3} \times \frac{3}{5} = \frac{6\cdot 4}{5} = 1\cdot 28$.

Consequently, the equation, (art. 265.) $\frac{FV}{l\sqrt{32\frac{1}{6}f\delta p}} =$

$$\frac{FV}{l\sqrt{32\frac{1}{6} \times 850 \times \frac{\cdot 04}{12} \times 1\cdot 28}} = \frac{FV}{l \times 10\cdot 8} = bd.$$

287. In beams supported at both ends, and of the same breadth, the power of a parabolic beam to resist a moving force, is to that of an uniform beam, as 10 is to 8 nearly; and the parabolic beam requires very little more than two-thirds of the quantity of material.

288. Rule. When F is the force in pounds, V its velocity in feet per second, l the whole length between the supports, and b and d the breadth and depth in inches; then $\frac{FV}{10\cdot 8l} = bd$.

289. Example. Let the force of a steam engine be applied to the middle of its beam, so as to cause

* Dr. Hutton's Course. vol. ii. p. 126.

† Idem. vol. ii. p. 327.

it to move an axis by means of two cranks, placed so as to be impelled by the ends of the beam. Let the greatest pressure on the piston be 3000 lbs. its greatest velocity three feet per second, and the whole length twelve feet.

By the rule, (art. 288.) $\frac{\mathrm{FV}}{10{\cdot}8l} = \frac{3000 \times 3}{10{\cdot}8 \times 12} = bd =$ 70 inches.

290. Case 2. When a parabolic beam rests upon a centre of motion, and the moving force acts at one end, and is opposed by a greater resistance at the other.

By art. 116. $f = 212$, and by art. 189. $\delta = \frac{{\cdot}16(1+r)}{12}$; also, $p = 32 \times \frac{2}{3} \times \frac{2}{5} = \frac{12{\cdot}8}{15} = {\cdot}853\dot{3}$; hence

$$\frac{\mathrm{FV}}{l\sqrt{32\frac{1}{6} f \delta p}} = \frac{\mathrm{FV}}{l\sqrt{32\frac{1}{6} \times 212 \times {\cdot}85\dot{3} \times \frac{{\cdot}16(1+r)}{12}}} =$$

$$\frac{\mathrm{FV}}{8{\cdot}82l\sqrt{1+r}} = b\,d.$$

291. Rule. Make F the force in pounds, and V its velocity in feet per second, l = the length in feet, from the centre of motion to the point where the force acts, and l' the length from the centre of motion to the resisted point; also, make b and d the breadth and depth in inches; then

$$\frac{\mathrm{FV}}{8{\cdot}82l\sqrt{1+\frac{l}{l'}}} = b\,d.$$

292. If $l = l'$, that is, when the centre of motion is in the middle of the beam, $\frac{FV}{12 \cdot 5l} = bd$.

293. In a steam engine the weight of the connecting apparatus, the power applied to the air-pump, and the weight of the catch-pins should be allowed for; and when the engine moves machinery, the beam should not be less than is determined by this rule. The depth of the beam is usually the same as the diameter of the steam-piston.

294. Example. If the pressure on the piston of a steam engine be 15,000 lbs., the whole length of the beam twenty-four feet, and its velocity three feet per second, required the area of the beam?

In this case, $\frac{FV}{12 \cdot 5l} = \frac{15{,}000 \times 3}{12 \cdot 5 \times 12} = bd = 300$ inches.

If the beam be made 48 inches deep, it should be $6\frac{1}{4}$ inches in breadth; and the best method of forming such a beam is to make it in two parts, each $3\frac{1}{4}$ in breadth, placed at 12 or 14 inches apart, and well connected together. This arrangement causes an engine to work with more steadiness, and the parts are less troublesome to move and fix in their places than a single mass would be.

295. Prop. 4. To determine a rule for finding the area of the middle section of a beam of uniform breadth, the depth at the end being half the depth in the middle, and the middle of the depth open, to resist a moving force.

Let the parts be so arranged that the centre of

gravity may be considered to be at the middle of the length of the arm of the beam, which will be very nearly true in practice, and will render the computation somewhat easier.

296. Case 1. When an open beam is supported at the ends, and the force is applied in the middle of the length.

By art. 162. $f = 850(1-p^3)$, and by art. 193. $\delta = \frac{\cdot 0327}{12}$; also, we have $p = \frac{3\cdot 2 \times (1-p')}{2} = 1\cdot 6(1-p')$; and the Equation, (art. 265.) $\frac{FV}{l\sqrt{32\frac{1}{6} f \delta p}} =$

$$\frac{FV}{l\sqrt{32\frac{1}{6} \times 850(1-p'^3) \times \frac{\cdot 0327}{12} \times 1\cdot 6(1-p')}} =$$

$$\frac{FV}{10\cdot 92 l\sqrt{(1-p'^3)\times(1-p')}} = b\,d.$$

297. Rule. Make F the force in pounds, V its velocity in feet per second, l the whole length between the supports in feet, p' that number which would be produced by dividing the depth of the part left out in the middle, by the whole depth; (if this ratio were not fixed the solution could not be effected;) and b and d the breadth and depth in the middle in inches, then $\frac{FV}{10\cdot 92 l\sqrt{(1-p'^3)\times(1-p')}} = bd.$

298. If p' be made $= \cdot 7$ which is a convenient proportion, then $\frac{FV}{4\cdot 85 l} = b\,d.$

299. Case 2. When an open beam is supported on

a centre of motion, and the moving force acts at one end, and the resistance at the other.

By the same method as above we find

$$\frac{FV}{10{\cdot}92l\sqrt{(1-p^3)\times(1-p)\times(1+r)}} = bd.$$

300. Rule. Make F = the force in pounds, V its velocity in feet per second, l the length from the point where the force acts to the centre of motion in feet, and l' the length from the centre of motion to the point of resistance, b and d the breadth and depth in inches in the middle of the beam, and p the number arising from the division of the depth of the part left out in the middle by the whole depth; then, $\dfrac{FV}{10{\cdot}92l\sqrt{(1-p^3)\times(1-p)\times(1+\frac{l'}{l})}}$

$= b\ d.$

301. If $p = {\cdot}7$ the equation reduces to

$$\frac{FV}{4{\cdot}85l\sqrt{1+\frac{l'}{l}}} = b\ d.$$

302. Also, when the centre of motion is in the middle of the beam, and $p = {\cdot}7$, we have $\dfrac{FV}{6{\cdot}86l} = bd.$

303. Example. As an example to the equation in the last article, let us suppose the pressure on the piston of a steam engine to be 15,000 lbs. its velocity three feet per second, and the whole length of the beam twenty-four feet, which is the same as the

example, (art. 294.) In this case, $\frac{FV}{6{\cdot}86l} = \frac{15{,}000 \times 3}{6{\cdot}86 \times 12}$ $= 771$ inches $= bd$.

And, if the depth be made forty-eight inches, then $\frac{771}{48} = 16{\cdot}06$ inches the breadth, which is to be the same throughout the length. The bulk of the metal in the upper and lower part of the beam will be found by multiplying the depth by $\cdot 7$; that is $\cdot 7 \times 48 = 33{\cdot}6$; which deducted from 48, leaves 14·4 inches, or 7·2 inches for each side.

Fig. 34. *Plate* IV. shows a sketch for a beam of this kind, drawn according to these proportions.

304. Any of the rules of this, or of the preceding Section, may be applied to other materials by substituting the proper values of the cohesive force, extensibility, and density; these are given for several kinds in the following table.

TABLE OF DATA, &c.

USEFUL IN VARIOUS CALCULATIONS;

ARRANGED ALPHABETICALLY.

The data correspond to the mean temperature and pressure of the atmosphere, dry materials; and the temperature is measured by Fahrenheit's scale.

AIR. Specific gravity 0·0012; weight of a cubic foot 0·0753 lbs. *(Shuckburgh)*; expands $\frac{1}{483}$ of its bulk by the addition of one degree of heat. *(Dalton.)*

ASH. Specific gravity 0·76; weight of a cubic foot 47·5 lbs.; will bear without permanent alteration a strain of 3540 lbs. upon a square inch = 0·23 times that of cast iron, and an extension of $\frac{1}{464}$ of its length = 26 times that of cast iron. *(Calculated from Mr. Barlow's Experiments.)*

ATMOSPHERE. Mean pressure of, at London, 28 89

inches of Mercury=14·18 lbs. upon a square inch. *(Royal Society.)*

Beech. Specific gravity 0·696; weight of a cubic foot 45·3 lbs.; will bear without permanent alteration on a square inch 2360 lbs.=0·15 times that of cast iron, and an extension of $\frac{1}{570}$ of its length=21 times the extension of cast iron. *(From Mr. Barlow's Experiments.)*

Brass, *cast.* Specific gravity 8·1; weight of a cubic foot 506¼ lbs.; expands $\frac{1}{93800}$ of its length by one degree of heat *(Troughton);* melts at 1869° *(Daniel);* cohesive force of a square inch 18,000 lbs. *(Rennie.)*

Brick. Specific gravity 1·841; weight of a cubic foot 115 lbs.; absorbs $\frac{1}{15}$ of its weight of water; cohesive force of a square inch 275 lbs. *(Tredgold);* is crushed by a force of 562 lbs. on a square inch. *(Rennie.)*

Brick-work. Weight of a cubic foot of newly built, 117 lbs. weight of a rod of new brick-work 16 tons.

Cast Iron. Specific gravity 7·207; weight of a cubic foot 450 lbs.; expands $\frac{1}{162000}$ of its length by one degree of heat *(Roy);* greatest change of length in the shade in this climate $\frac{1}{1723}$; greatest change of length exposed to the sun's rays

$\frac{1}{1270}$; melts at 3479° *(Daniel)*; and shrinks in cooling from $\frac{1}{98}$ to $\frac{1}{85}$ of its length *(Muschet)*; will bear without permanent alteration 15,300 lbs. upon a square inch, and an extension of $\frac{1}{1204}$ of its length *(Tredgold)*; is crushed by a force of 93,000 lbs. upon a square inch. *(Rennie.)*

CHALK. Specific gravity 2·315; weight of a cubic foot 144·7 lbs. is crushed by a force of 500 lbs. on a square inch. *(Rennie.)*

CLAY. Specific gravity 2·0; weight of a cubic foot 125 lbs.

COAL. Specific gravity 1·269; weight of a cubic foot 79·31 lbs. A London chaldron of 36 bushels, weighs about 28 cwt. A Newcastle chaldron, 53 cwt. *(Smeaton.)*

COPPER. Specific gravity 8·75 *(Hatchett)*; weight of a cubic foot 549 lbs.; expands in length by one degree of heat $\frac{1}{105900}$ *(Smeaton)*; melts at 2548° *(Daniel)*; cohesive force of a square inch, when hammered, 33,000 lbs. *(Rennie.)*

EARTH, *common*. Specific gravity 1·52 to 2·00; weight of a cubic foot from 95 to 125 lbs.

ELM. Specific gravity 0·544; weight of a cubic foot 34 lbs. will bear on a square inch without permanent alteration 3240 lbs. $=$ 0·21 times cast iron; and an extension in length of $\frac{1}{414}$ $=$ 29 times that of cast

iron. *(Calculated from Mr. Barlow's Experiments).*

FIR, *red or yellow.* Specific gravity 0·557; weight of a cubic foot 34·8 lbs.; will bear on a square inch without permanent alteration 4290 lbs.=0·3 times cast iron, and an extension in length of $\frac{1}{470}$=26 times that of cast iron. *(Tredgold.)*

FIR, *white.* Specific gravity 0·47; weight of a cubic foot 29·3 lbs.; will bear on a square inch without permanent alteration 3630 lbs.= 0·23 times cast iron, and an extension in length of $\frac{1}{504}$=24 times that of cast iron. *(Tredgold.)*

FLOORS. The weight of a superficial foot of a floor is about 40 lbs. when there is a ceiling, counterfloor, and iron girders. When a floor is covered with people, the load upon a superficial foot may be calculated at 120 lbs. Therefore 120+40=160 lbs. on a superficial foot, may be used in estimating the greatest load a floor of a room has to sustain.

FORCE. See Gravity, Horses, &c.

GRANITE, *Aberdeen.* Specific gravity 2·625; weight of a cubic foot 164 lbs. is crushed by a force of 10,910 lbs. upon a square inch. *(Rennie.)*

GRAVITY, generates a velocity 32$\frac{1}{6}$ feet, in a second, in a body falling from rest; space described in the first second 16$\frac{1}{12}$ feet.

HORSE, produces the greatest effect when exerting a force of 200 lbs. with a velocity of 3$\frac{2}{3}$ feet per

second, working eight hours in a day. A good horse can exert a force of 480 lbs. for a short time. *(Desaguliers.)*

IRON, *cast*. See Cast Iron.

IRON, *malleable*. Specific gravity 7·6 *(Muschenbroek)*; weight of a cubic foot 475 lbs. ditto when hammered 487 lbs.; expands in length, by 1° of heat $\frac{1}{143,000}$ *(Smeaton)*; good English iron will bear on a square inch without permanent alteration 17,800 lbs.=1·12 times cast iron, and an extension in length of $\frac{1}{1400}$=0·86 times that of cast iron. *(Tredgold.)*

LEAD, *cast*. Specific gravity 11·352, *(Brisson)*; weight of a cubic foot 709·5 lbs. expands in length by 1° of heat $\frac{1}{62800}$ *(Smeaton)*; melts at 612° *(Crichton)*; will bear on a square inch without permanent alteration 1,500 lbs.=0·096 times cast iron, and an extension in length of $\frac{1}{480}$=25 times that of cast iron. *(Tredgold.)*

MAHOGANY, *Honduras*. Specific gravity 0·56; weight of a cubic foot 35 lbs.; will bear on a square inch without permanent alteration 3800 lbs.=0·24 times cast iron, and an extension in length of $\frac{1}{420}$=29 times that of cast iron. *(Tredgold.)*

MAN, produces the greatest effect when exerting a

force of 80 lbs. with a velocity of $3\frac{1}{2}$ feet per second, for 10 hours in a day. A strong man will raise and carry from 250 to 300 lbs. (*Desaguliers.*)

MARBLE, *white.* Specific gravity 2·706; weight of a cubic foot 169 lbs. cohesive force of a square inch 1,811 lbs.; extensibility $\frac{1}{1394}$ of its length (*Tredgold*); is crushed by a force of 6,060 lbs. upon a square inch. (*Rennie.*)

MERCURY. Specific gravity 13·568 (*Brisson*); expands in bulk by one degree of heat $\frac{1}{9990}$. (*Dulong and Petit.*)

OAK, *good English.* Specific gravity 0·83; weight of a cubic foot 52 lbs.; will bear upon a square inch without permanent alteration 3960 lbs. = 0·25 times cast iron, and an extension in length of $\frac{1}{433}$ = 28 times that of cast iron. (*Tredgold.*)

PENDULUM. Length of pendulum to vibrate seconds in the latitude of London 39·1372 inches (*Kater*); ditto to vibrate half seconds 9·7843 inches.

PINE, *American, yellow.* Specific gravity 0·46; weight of a cubic foot $26\frac{1}{2}$ lbs. will bear on a square inch without permanent alteration 3900 lbs. = 0·25 times cast iron, and an extension in length of $\frac{1}{414}$ = 29 times that of cast iron. (*Tredgold.*)

PORPHYRY. Specific gravity 2·871; weight of a cubic

foot 179 lbs. is crushed by a force of 35,568 lbs. upon a square inch. (*Gauthey.*)

ROOFS. Weight of a square foot of Welsh rag slating $11\frac{1}{4}$ lbs.; weight of a square foot of plain tiling $16\frac{1}{4}$ lbs.; greatest force of the wind upon a superficial foot of roofing may be estimated at 40 lbs.

SLATE, *Welsh.* Specific gravity 2·752 (*Kirwan*); weight of a cubic foot 172 lbs. cohesive force of a square inch 12,800 lbs. (*Tredgold.*)

STEAM. Specific gravity at 212° is to that of air at the mean temperature as 0·472 is to 1. (*Thomson.*)

STEEL. Specific gravity 7·84; weight of a cubic foot 490 lbs.; expands in length by 1° of heat $\frac{1}{157,200}$ (*Roy*); cohesive force of a square inch 130,000 lbs. (*Rennie.*)

STONE, *Portland.* Specific gravity 2·113; weight of a cubic foot 132 lbs.; absorbs $\frac{1}{16}$ of its weight of water (*R. Tredgold*); cohesive force of a square inch 857 lbs.; extends before fracture $\frac{1}{1789}$ of its length (*Tredgold*); is crushed by a force of 3,729 lbs. upon a square inch. (*Rennie.*)

STONE, *Bath.* Specific gravity, 1·975; weight of a cubic foot 123·4 lbs.; absorbs $\frac{1}{13}$ of its weight of water (*R. Tredgold*); cohesive force of a square inch 478 lbs. (*Tredgold.*)

STONE, *Craigleith.* Specific gravity 2·362; weight of a

cubic foot 147·6 lbs.; absorbs $\frac{1}{63}$ of its weight of water; cohesive force of a square inch 772 lbs. (*Tredgold*); is crushed by a force of 5,490 lbs. upon a square inch. (*Rennie.*)

STONE, *Dundee.* Specific gravity 2·621; weight of a cubic foot 163·8 lbs.; absorbs $\frac{1}{511}$ part of its weight of water; cohesive force of a square inch 2,661 lbs. (*Tredgold*); is crushed by a force of 6,630 lbs. upon a square inch. (*Rennie.*)

STONE-WORK. Weight of a cubic foot about 140 lbs.

TIN, *cast.* Specific gravity 7·291 (*Brisson*); weight of a cubic foot 455·7 lbs.; expands in length by 1° of of heat $\frac{1}{72,510}$ (*Smeaton*); melts at 442° (*Crichton*); will bear upon a square inch without permanent alteration 2,880 lbs.=0·182 times cast iron, and an extension in length of $\frac{1}{1600}$=0·75 times that of cast iron. (*Tredgold.*)

WATER. Specific gravity 1·000; weight of a cubic foot 62·5 lbs. weight of an ale gallon of water 10·2 lbs.; expands in bulk by one degree of heat $\frac{1}{3858}$ (*Dalton*);* expands in freezing $\frac{1}{17}$ of its bulk

* Water has a state of maximum density, at or near 40°; which is considered an exception to the general law of expansion by heat; it is extremely improbable that there is any thing more than an ap-

(*Williams*); and the expanding force of freezing water is about 35,000 lbs. upon a square inch, according to Muschenbroek's valuation.

WIND. Greatest observed velocity 159 feet per second (*Rochon*); force of wind with that velocity about $57\frac{1}{4}$ lbs. on a square foot*.

ZINC, *cast.* Specific gravity 7·028 (*Watson*); weight of a cubic foot $439\frac{1}{4}$ lbs.; expands in length by one degree of heat $\frac{1}{61200}$ (*Smeaton*); melts at 648° (*Daniel*); will bear on a square inch without permanent alteration 5,700 lbs.=0·365 times cast iron, and an extension in length of $\frac{1}{2400}=\frac{1}{2}$ that of cast iron. (*Tredgold.*)†

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parent exception, most likely arising from water at low temperatures absorbing a considerable quantity of air, which has the effect of expanding it; and consequently of causing the apparent anomaly.

* Table of the force of winds formed from the tables of Mr. Rouse and Dr. Lind.

A wind may be denominated when it does not exceed the velocity opposite to it,	Velocity per second.	Force on a square foot.
A gentle pleasant wind	10 feet.	0·129 lbs.
A brisk gale	20 —	0·915 —
A very brisk gale	30 —	2·059 —
A high wind	50 —	5·718 —
A very high wind	70 —	11·207 —
A storm or tempest	80 —	14·638 —
A great storm	100 —	22·872 —
A hurricane	120 —	32·926 —
A violent hurricane, that tears up trees, overturns buildings, &c.	150 —	51·426 —

† The fracture of zinc is very beautiful; it is radiated, and preserves its lustre a long time.

Note on the Action of certain Substances on Cast Iron.

In some circumstances, cast iron will decompose, and be converted into a soft substance resembling plumbago. Some instances of this kind I add here, as they will be interesting to persons who employ iron for various purposes.

Dr. Henry observed that when cast iron was left in contact with muriate of lime, or muriate of magnesia, most of the iron was removed, the specific gravity of the mass was reduced to 2·155, and what remained consisted chiefly of plumbago, and the impurities usually found in cast iron.*

A similar change was produced in some cast-iron cylinders used to apply the weaver's dressing to cloth; this dressing is a kind of paste made of wheat or barley meal. The corrosion of the cylinders took place repeatedly, and was so rapid that it was found necessary to use wooden ones. Dr. Thomson ascribes the change to the acid formed by the paste turning sour.†

Another instance, of greater importance, has been recorded by Mr. Brande. A portion of a cast-iron gun had undergone a like change from being long immersed in sea-water. To the depth of an inch it was converted into a substance, having all the external characters of plumbago; it was easily cut, greasy to the feel, and made a black streak upon paper.‡

* Dr. Thomson's Annals of Philosophy, vol. v. p. 66. † Idem, vol. x. p. 302.
‡ Quarterly Journal of Science, vol. xii. p. 407.

EXPLANATION

OF THE

PLATES.

PLATE I.

FIG. 1. A bar supported at the ends, and loaded in the middle of the length. See art. 7.

FIG. 2. A beam, with the load uniformly distributed over the length, as the experiment, art. 50. was tried. See art. 18, and 50.

FIG. 3. The form for a beam of uniform strength to resist the action of a load at C. ACD, and BCD, are semi-parabolas, A and B being the vertices. The dotted lines show the additions to this form to render it of practical use. See art. 22, 89, and 185—191.

FIG. 4. A form for a beam which is as nearly of uniform resistance as practical conditions will admit of; it is bounded by straight lines, and the depths at the ends are each half the depth in the middle. See art. 23, 54, and 192—196.

FIG. 5. A variation of the last form for the case where the force sometimes acts upwards and sometimes downwards. See art. 24, and 192—196.

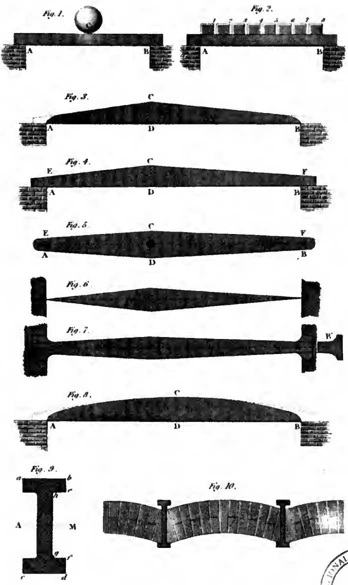
FIG. 6. A figure of uniform strength for a beam, when the depth is uniform. See art. 25, 88, and 204—208.

FIG. 7. A modification of *Fig.* 6. which is the most economical form of equal strength for resistance to pressure. B′ is the form of the end. See art. 25.

FIG. 8. The form of equal resistance for a load rolling along the upper side, as in a railway; or for a load uniformly distributed over the length. ACB is half an ellipse. The dotted lines show the addition required in practice. See art. 26, 92, 202, and 203.

FIG. 9. The strongest form of section for a beam to resist a cross strain. AM is the line called the *neutral axis*. See art. 31, 43, 84, 147—159, and 274.

FIG. 10. Shows an application of the section *Fig.* 9. to form a fire-proof floor, the projection serving the double purpose of giving additional strength, and forming a support for the arches. See art. 31, and 156.



Drawn by Thos Tredgold

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M

PLATE II.

Fig. 11. This is the figure of a very economical beam for supporting a load diffused over its length; it is adapted for girders, beams to support walls, and the like. When used for a girder the openings answer for inserting cross joists. AB and CD show the sections at these places. See art. 32, 34, 85, 160—172, and 276.

Fig. 12. This figure shows a beam on the same principle as the preceding figure, except that the load is supposed to act only at one point A. See art. 34, 85, and 160—172.

Fig. 13. The section of a shaft, commonly called a feathered shaft. See art. 35.

Fig. 14. A figure to illustrate the action of forces upon a beam, and to explain the mode of calculation. See art. 75, 77, 94, and 117.

Fig. 15. A section of the beam in *Fig.* 14. at BD. This section is supposed to be divided into thin laminæ. See art. 75.

Fig. 16. A figure to illustrate the method of calculating the deflexion of beams. In these figures (*Fig.* 14 and 16.) I have regarded distinctness of the parts referred to, more than the true relation of the parts to one another. See art. 86.

PLATE 11

Fig 11

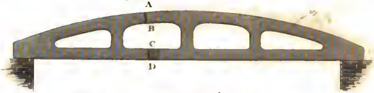

Fig 12

Fig 13

Fig 14

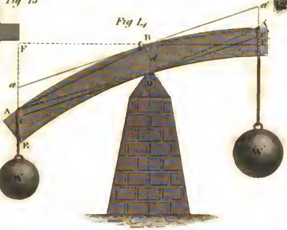

Fig 13

Fig 16

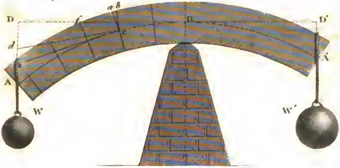

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PLATE III.

Fig. 17. Is to illustrate the circumstances which take place in the deflexion of beams fixed at one end. See art. 96, and 117.

Fig. 18. To explain the mode of calculating the strength of cranks. See art. 98.

Fig. 19. A figure to explain the manner of estimating the strength and deflexion of a beam supported at the ends. See art. 99, 106, 109, 112, and 127.

Fig. 20. A figure to show how to calculate the strain upon a beam when a load is distributed in any regular manner over it. The load being uniform *a d* is the line which represents its upper surface; when the load increases as the distance from the end A, *c d* is the line bounding it; and when the load increases as the square of the distance from A, *bd* is the line bounding it. The second case is the same as the pressure of a fluid against a vertical sheet fixed at the ends. See art. 101—104, and 122.

Fig. 21. ACB is the form of equal strength for an uniform load; it is in this figure applied to the cantilever of a balcony, and whatever ornamental form may be given to the under side, it should not be cut within the line BC. See art. 120.

Fig. 22. When the section is of the form C′ D′, and the breadth uniform, the figure of equal strength for a load in the middle is formed by two semi-parabolas (as in *Fig.* 3.) shown by the dotted lines; and it may be formed for practical application as shown in the figure. See art. 149, 185—189.

Fig. 23. Is a figure to illustrate the nature of variable forces. See art. 249.

Fig. 24. If the section of the beam be C′ D′, and the breadth uniform, the form of equal strength for an uniform load is a semi-ellipse, shown by the dotted lines; and also when the load rolls or slides over it; and it may be formed for practical application as the figure. See art. 150, 155, 202, 203, and 275.

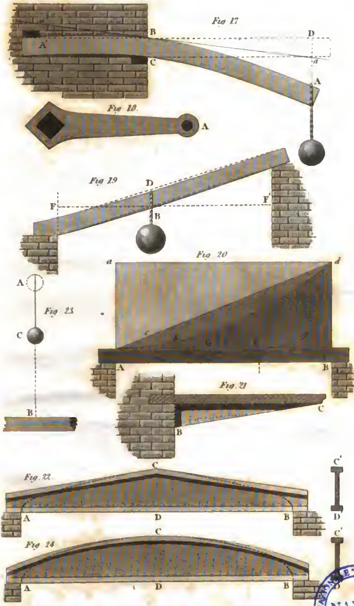

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PLATE IV.

Fig. 25. Represents a beam fixed at one end; $a' b'$ is its section; the load acting at the end C, the figure of equal resistance is a semi-parabola. See art. 158, and 185—191.

Fig. 26. Form suitable for the beam of a steam engine to the form of section $a' b'$. See art. 158, 184, and 185—191.

Fig. 27. A sketch for a beam to bear a considerable load distributed uniformly over its length, when the span is so much as to render it necessary to cast it in two pieces. The connection may be made by a plate on each side at C, with indents to fit the corresponding parts of the beam. See the next figure, and art. 160—172.

Fig. 28. Shows the under side of the beam, in the preceding figure; the plates are held together by bolts; but, it is intended that the strength should depend on the indents, the bolts being only to hold them together; no connection is wanted at the upper side of the beam, except a bolt $c\,d$, or like contrivance, to steady it. See art. 161.

Fig. 29. A figure to explain the nature of the resistance to twisting, or torsion. See art. 223.

Fig. 30. A figure to illustrate the action of the straining force on columns, posts, and the like. See art. 230.

Fig. 31. To explain the effect of settlement, or other derangement of the straining force. See art. 235.

Fig. 32. Another case of settlement or derangement of the straining force on a column considered. See art. 237.

Fig. 33. To show why columns should not be enlarged at the top or bottom. See art. 237.

Fig. 34. A sketch for an open beam recommended for an engine beam. See art. 303. In small beams the middle part may be left wholly open except at the centre. Capt. Kater has used this form for the beam of a delicate balance.

Fig 25

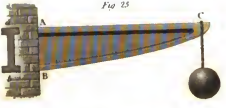

Fig 29

Fig. 26.

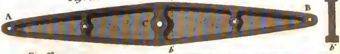

Fig 27.

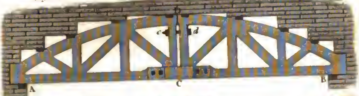

Fig 28.

Fig 30

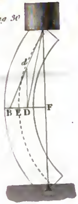

Fig 31

Fig 32.

Fig 33

Fig 34

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A LIST OF AUTHORS

Quoted in the Alphabetical Table, with the Titles of the works from which the Data have been quoted.

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INDEX,

WITH

COPIOUS REFERENCES TO THE PRACTICAL RULES.

The Figures refer to the Pages.

THE END.

ERRATA.

PAGE.	
44	Line 8 *for* *bb* *read* *ab*.
48	Line 12 *for* deflexions *read* deflexion.
66	Line 4 *for* : l *read* 1 : l.
70	Line 4 *for* $(1\frac{1}{-n^4})$ *read* $(1-\frac{1}{n^4})$.
71	Line 8 *for* ABA *read* ABA′.
72	Line 2 *dele* 8.
76	Line 5 from the bottom, *for* deflexions *read* deflexion.
78	Line 4 *dele* 8.
88	Line 2 *for* $\frac{Wl}{4}7854fr^3$ *read* $\frac{Wl}{4}=\cdot 7854fr^3$.
89	Line 10 *for* 30 *read* 130.
117	Bottom line add a comma after $+AD^2$ and the sign of multiplication between AD BD.
126	In Equation xxxiv. *read* $6a+\frac{\cdot 18\,l^3}{d}$ *instead of* $ba+\frac{\cdot 18\,l^3}{d}$.
134	Line 6 from the bottom *for* $\frac{\sqrt{32PS}}{W}$ *read* $\frac{\sqrt{32\frac{1}{6}PS}}{W}$.
140	Line 7 *for* support *read* supports.
144	Bottom line *for* $\frac{l}{l}$ *read* $\frac{l'}{l}$.

J. and T. Bartlett, Printers, Oxford.